# Metodologia. Exercises

## Code 1

```
1) j = n        } 2
2) k = 1
3) While j ≥ do     } 1  }
4)     k = k + 1    } 2  } 5
5)     j = n/k      } 2  }
6) fin_while        } 1
```

$$T_1(n) = 2 + T_{while} = 2 + \sum_{k=1}^{n} 5 + 1 = 3 + 5n$$

## Code 2

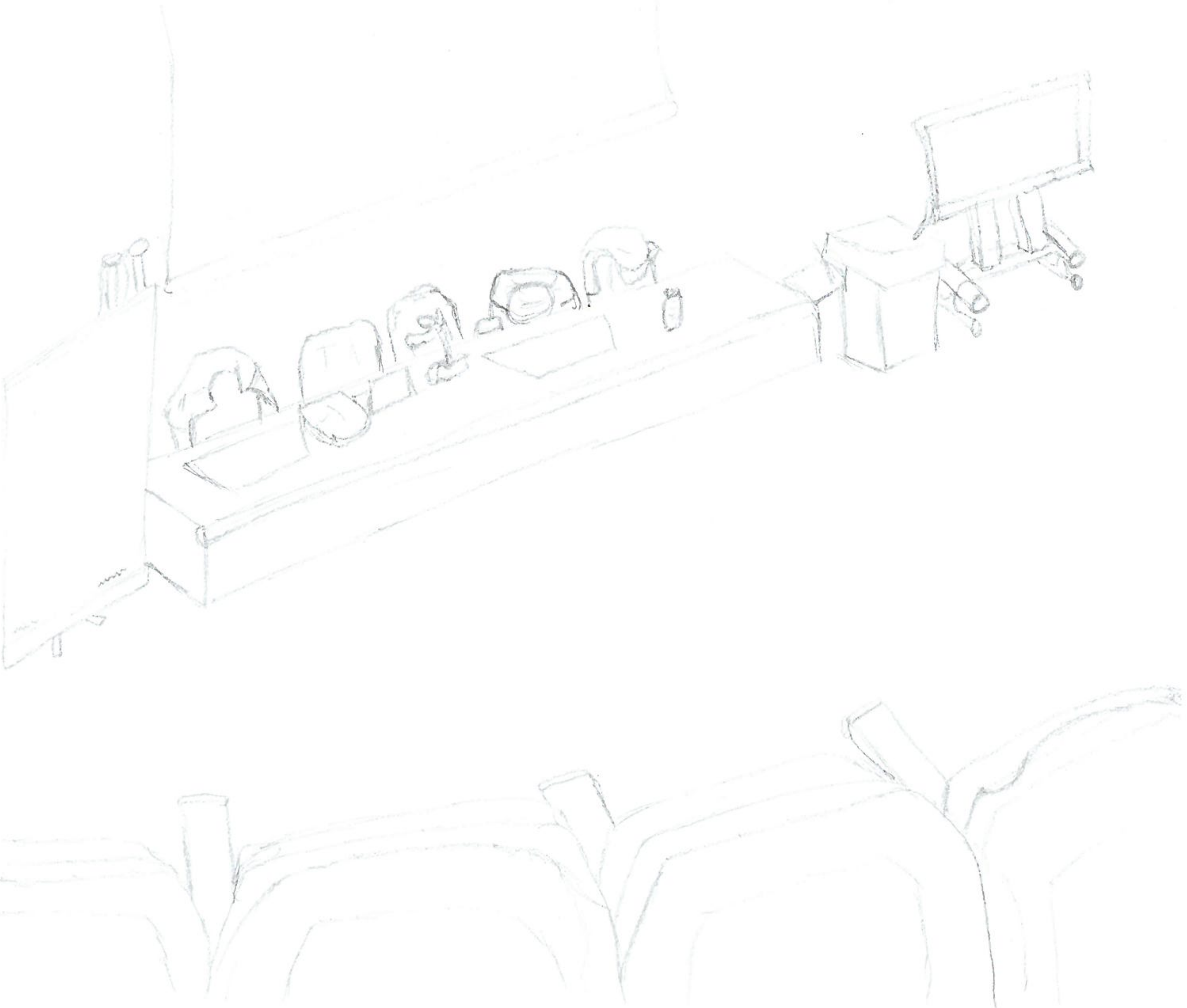
```
1) j = n
2) while j ≥ 1 do   } 1
3)     j = j/5      } 2
4) fin_while
```

last value answered as yes $(5^0 = 1)$

$$T_2(n) = 2 + \sum_{j=n}^{1} 3 \overset{j=5^i}{=} 2 + \sum_{i=\log_5(n)}^{\boxed{0}} 3 = 2 + 3(\log(n) + 1) = = 5 + 3\log_5(n)$$

$$\parallel$$

$$2 + 3(\log_5(n) + 1)$$



```
               → 1 < b-a
while (a<b) } k=1, t=1;                  (1)
   for (i=1, m, i++) } t=t+m(*); {       (2)
   while (k<t) } k=k+2;                  (3)
        for (i=1, k, i++) } c++{         (4)
        {
   do } k=k/3; { while (k >0);           (5)
```

$$^{(1)}\sum_{m}^{1}\left(5+{}^{(2)}\sum_{i=1}^{m}5+{}^{(3)}\sum_{k=1}^{t}\left(4+{}^{(4)}\sum_{i=1}^{k}5\right)+{}^{(5)}\sum_{k}^{0}3\right)$$

$$\sum_{m}^{1}{}_{1}\left(5\sum_{i=1}^{m}{}_{2}5+\sum_{k=1}^{m^2}{}_{3}\left(4+\sum_{i=1}^{k}{}_{4}5\right)+\sum_{k}^{0}{}_{5}3\right.$$

$$\sum_{m}^{1}{}_{1}\left(5\sum_{i=1}^{m}{}_{2}5+\sum_{k=1}^{m^2}\left(4+\sum_{i=1}^{k}{}_{4}5\right)+\sum_{m^2}^{0}{}_{5}3\right.$$

$\textcircled{t}$ (m+m+m+m)
m veces(*) $(m\cdot m=m^2)$

k (2+2+2+2)
$\frac{m^2}{2}$ veces $k=m^2$

| Sumatorio | Bucle | Tamaño | Big O |
|---|---|---|---|
| $\Sigma_1$ | while (1<m) | m/2 | $m^5$ |
| $\Sigma_2$ | for(i) | m | m |
| $\Sigma_3$ | while(k<t) | $m^2/2$ | $m^4$ |
| $\Sigma_4$ | for (i, k) | $m^2$ | $m^2$ |
| $\Sigma_5$ | do while | $\log_3(m^2)$ | $\log_3(m^2)$ |

$$\sum_{m}^{1}\left(5+\sum_{i=1}^{m}5+\sum_{k=1}^{m^2}\left(4+\sum_{i=1}^{k}5\right)+3\log_3(m^2)\right)$$

$$\sum_{m}^{1}\left(5+\sum_{i=1}^{m}5+\sum_{k=1}^{m^2}(4+5k)+3\log_3(m^2)\right)$$

$$\sum_{m}^{1}\left(5+\sum_{i=1}^{m}5+4m_2+5m^{2^2}+3\log_3(m^2)\right)$$

$$\sum_{m}^{1}\left(5+5m+4m^2+5m^4+3\log_3(m^2)\right)$$

$$5m+5m^2+4m^3+5m^5$$

ice: $m : b-a$

$$\sum^{m} \Big( 5 \qquad + \sum_{k}^{0} 3$$

$$\sum_{k}^{t} 3 + \sum_{i=1}^{2m/k} 5 \Big)$$

$$\sum_{m}^{n} 1/t \Rightarrow \int 1/t = \lg \ln$$

Coins: } 1,2,2,2,5,7,10 {     Max: 24

"Mínimo que devolvemos de monedas"

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | ∞ | | ... | | | | | | | ... | | | | ... | | | | | ... | | | | | ∞ |
| 1 | 0 | 1 | ∞ | | ... | | | | | | ... | | | | ... | | | | | ... | | | | | ∞ |
| 1,2 | 0 | 1 | 1 | 2 | ∞ | | | | | | ... | | | | ... | | | | | ... | | | | | ∞ |
| 1,2,2 | 0 | 1 | 1 | 2 | 2 | 3 | ∞ | | | | | | | | | | | | | | | | | | ∞ |
| 1,2,2,2 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | ∞ | | | | | | | | | | | | | | | | |
| 1,2,2,2,5 | 0 | 1 | 1 | 2 | 2 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | ∞ | | | | | | | | | | | |
| 1,2,2,2,5,7 | 0 | | | | | | | | | | | | | | | | | | | ∞ | | | | | |
| 1,2,2,2,5,7,10 | 0 | | | | | | | | | | | | | | | | | | | | | | | | |

→ $M(i,j)$

"minimum # coins from set "i" to refund "j""

$m(i,0)=0; m(i,j)=m(i-1,j) \quad value(i^*) j$

Candidates: coins

$$\min \} M(i-\$,j), M(i-1,j-value(i^*))+1 \{$$

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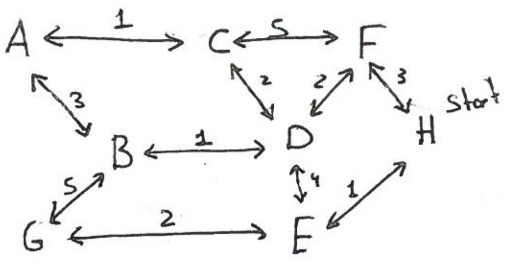
1/



Greedy

| start vertex | F | E | C | D | B | G | A |
|---|---|---|---|---|---|---|---|
| E | 3 | ① | ∞ | ∞ | ∞ | ∞ | ∞ |
| F | ③ | ① | ∞ | 5 | ∞ | 3 | ∞ |
| G | ③ | ① | 8 | ⑤ | 6 | ③ | ∞ |
| D | ③ | ① | 7 | ⑤ | ⑥ | ③ | 8 |
| B | | | | | | | |
| C | | | | | | | |
| A | | | | | | | |

$\underbrace{\quad}_{Sum = 33}$

Sum = 33

Candidates: Arch/edges

Order: increasing order

Feasibility: if $(d[v] \notin RN)\}$

$if(d[u] + c(u,v) < d[v])\}$

$d[v] = d[u] + c(u,v);$

$\{else\}$ $d[v] = d[v]$

Finish: All arcs/nodes checked

2/ $v_1, v_2 \ldots v_n$ { Best possible change in each case to paid m=21, I give 40 = M   M-m = 40-21 = 19

$v_1 = 2, 20, 5, 10$

| i \ j | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| }{ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| }2{ | 0 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| }2,20{ | 0 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| }2,20,5{ | 0 | 1 | 0 | 1 | 2 | 0 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| }2,20,5,10{ | | | | | | | | | | | | | | | | | | | | |

Heads of columns: M-m

Heads of rows: Sets of numbers

$C(i,j) = j$ $(i == 0)$

$C(i,j) = \min\}M[i-1, j]\, i^{*}-j\{$ $(i > j)$

$C(i,j) = \min\}M[i-1, j], M[i-1, j-i\{$ (o.w)

3) Increasing array, non-repeated numbers, subsets that sum up is multiple of 10
S = [-10, -3, -1, 0, 2, 4, 6, 13]

Backtracking strategy

Exhaustivity: B(i) ranges: 0 (not set) 1 (not included) 2 (included) 3 (BT)
BT Node => if B(i)=0 ; i--; if (B(i)==2)} Sum=Sum - S(i){
Dead node => (i==last) && ((Sum+[(B(i)== ?S(i):0] %10)= != 10
Live node => (Dead node)' \ if (B(i)==2)} Sum=Sum+S(i); {
Solution node => (i==last) && Live node) ; N++ ; if (B(i)==2)} Sum=Sum-S(i)

# Greedy Algorithm

```
Greedy (a, n) }
  for i:=1 to n do
    x:=select (a);
    if feasible (x) then}
      solution = solution +x; {{
```

n=5

a

| | | | | |
|---|---|---|---|---|
| $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
| 1 | 2 | 3 | 4 | 5 |

## → Knapsack Problem

n=7

m=15

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Objects (o): | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Profits (p): | 10 | 5 | 15 | 7 | 6 | 18 | 3 |
| Weights (w): | 2 | 3 | 5 | 7 | 1 | 4 | 1 |
| related (P/w) | 5 | 1'3 | 3 | 1 | 6 | 4'5 | 3 |

→ we apply the sort algorithm first

↳ 6, 5, 4'5, 3, 3, 1'3, 1

Candidates: Objects to be introduce

Order: Decreasing profits/weights

Feasibility: (int related, item): if(capacity ≥ item (weight ))
included it (value += item)
capacity -= item (weigth)

else} accepting ( capacity/item(weight)

End: (Capacity == 0)

15 - 1 = 14
14 - 2 = 12
12 - 4 = 8
8 - 5 = 3
3 - 1 = 2
$\boxed{2-3=-1}$ → else} (2/3 of 1'3)

WEIGHTS

## → Job Sequencing Deadlines

| Jobs | $J_1$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ | $J_6$ | $J_7$ |
|---|---|---|---|---|---|---|---|
| Profits | 35 | 30 | 20 | 16 | 15 | 18 | 5 |
| Deadlines | 3 | 4 | 4 | 2 | 3 | 1 | 2 |

→ Each job only takes 1ut.

Candidates: Jobs to be performed

Order: Decreasing profits

Feasibility: —

End: Total == MAX

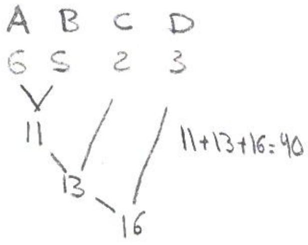
0 $\underline{J_6}$ 1 $\underline{J_3}$ 2 $\underline{J_1}$ 3 $\underline{J_2}$ 4

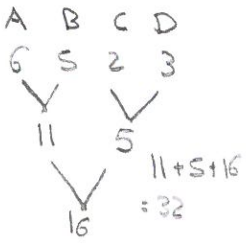
| Jobs | 0 | 1 | 2 | 3 | 4 | TOTAL |
|---|---|---|---|---|---|---|
| Ø | | | | | | Ø |
| $J_1$ | | | | $J_1$ | | 35 |
| $J_2$ | | | | $J_1$ | $J_2$ | 65 |
| $J_3$ | | | $J_3$ | $J_1$ | $J_2$ | 85 |
| $J_4$ | | $J_4$ | $J_3$ | $J_1$ | $J_2$ | 101 |
| $J_5$ x | | $J_4$ | $J_3$ | $J_1$ | $J_2$ | 101 |
| $J_6$ | | $J_6$ | $J_3$ | $J_1$ | $J_2$ | 103 |
| $J_7$ x | | $J_6$ | $J_3$ | $J_1$ | $J_2$ | 103 |

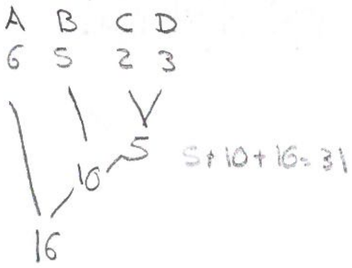
→ Best solution

## → Optical Merge Pattern

| | | | | |
|---|---|---|---|---|
| List → | A | B | C | D |
| Sizes → | 6 | 5 | 2 | 3 |

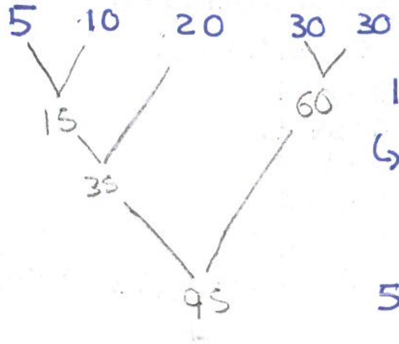






| | | | | | |
|---|---|---|---|---|---|
| List → | A | B | C | D | E |
| Size → | 20 | 30 | 10 | 5 | 30 |

Candidates: List of sizes
Order: Increasing values
Feasibility –
End: Total == MAX

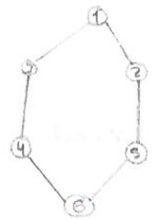


15+35+65+95=205

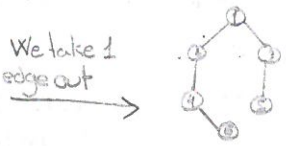
↳ Otra forma
↳ Cada línea hacia el final es "X" × num

5x3 + 10x3 + 2x20 + 2x30 + 2x30 = 205

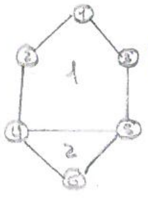
## → Minimum cost spanning tree



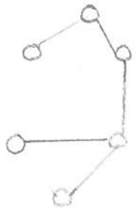
G=(V,E)
V={1,2,3,4,5,6{
E={(1,2),(2,3)...{



$6C_5 = 6$ ← Depends on the no of circles



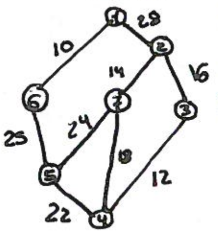
$|E| = 6$



$7C_5 - 2 = 5$

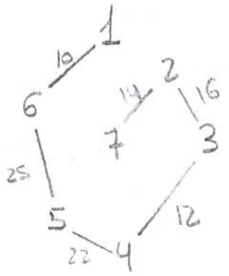
$^{|E|}C_{|V|-1} - $ no of cycles

## → Prim's



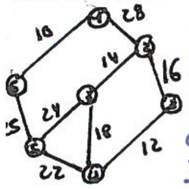
1- Coge el menor
2- Ve siguiendo por el que menos cueste unido

Cost = 99



$\Theta(|V| \cdot |E|)$
$\Theta(n \cdot e) = \Theta(n \cdot n) = \Theta(n^2)$

## → Kruskal's



1- Ve cogiendo el menor y aumenta
2- No hace falta que estén unidos
3- NO hacer círculos

Min heap → mínimo coste si fuera directo
$\Theta(n \cdot \log n) \rightarrow \Theta(V \cdot \log n)$

~> compute only once

## Supermarket

Candidates: Coins to be refounded

Order: Decreasing value

Feasibility: int amount= Money paid-bill;
} Refand : amant "DIV" Value(i);
amant : amant % amant; {

End/Finish: (Refand==0)

| Am | Refand | |
|---|---|---|
| 497 | 0 | 500 |
| 97 | 2 | 200 |
| 97 | 0 | 100 |
| ⋮ | ⋮ | ⋮ |

Greedy= The best, "ASAP"

## Knapsack

Candidates: Items to be introduced

Order: Decreasing value/weight

Feasibility:(int capacity, item)=if(capacity ≥ item(weight))
Included it (value+= item(value))
capacity -= item(weight)
else} accepting($\frac{\text{capacity}}{\text{item(weight}}$)

1° → adding pedia un int
(contador)

2° → boolean true/false

End/Finish: (Capacity== 0) ≡ Else

## Array

Candidates: Pairs of indices of the array [0] and [3]

Sorting Creteria: i:0  Neg=0; Pos=0

Feasibility: if (A(i)<0) Neg++; if(A(i)>0) Pos ++; i++

End: (i==last); Return ((Neg DIV 2) + (Pos DIV2))// Return ((Pos > Neg) ? Neg : Pos)

Cond ? exp1 : exp2

## Array IV

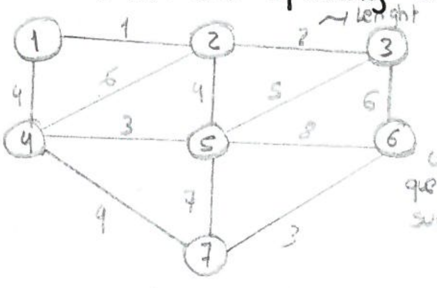
Candidates Indices of the given array

Sorting criteria: i:0 ; zeros=0

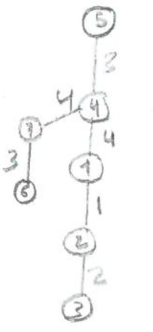
Feasibility if( A(i)==0) Zeros ++; i++;

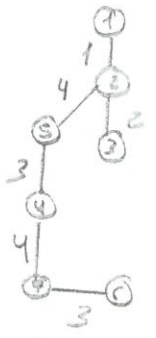
End: (i==last); Return((zeros>(n DIV2)?(N DIV2): Zeros;)

## Minimum Spanning Tree

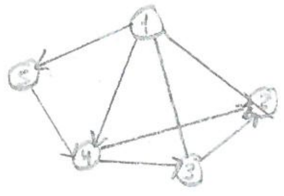


Kruskal
↳ Eliminar uno a uno los más largos sin que se quede un nodo suelto





## Shortest path from 1: DIJKSTRA



# Bellman optimality

For knapsack

↳ Setsof condidates

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| }items{ | 0 | 3 | 7 | 8 | 8 | 10 | 13 |
| }itms, skrawdnuel{ | | | | | | | |

| | Vale | weigth |
|---|---|---|
| items 1 | 4 | 7 |
| 2 | 3 | 8 |
| 3 | 5 | 9 |
| 4 | 4 | 8 |
| 5 | 5 | 10 |

Capacity 16  Items conot be divided

4
7

Si se hace milad asi pega de dose le dio la vuelta a value y weigth

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ∅ | 0 | 0 | 0 | . | . | . | | | | | | | | | | | 0 |
| }1{ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| }1,2{ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 4 | 4 | 4 | 4 | 4 | | | | |
| }1,2,3,1 | | | | | | | | | | | | | | | | | |

# Partition of two subsets of equal sum

$\boxed{1|2|3} \rightarrow \boxed{1|2} \quad \boxed{3}$

- Data: One-dim array 1000 elements / S(i)::j means "ith" element goes to "j" subset
- Exhaustivity: j ∈ }0 → 1 → 2 → 3{ $\sum_{i=1}^{1000} A(i) = 2B$ (Debe ser par) ; P = 0

(2, -5, 0, 6, 3, -1, 1) → B = 3 ⇒ 6elementos = 2B, B=3

¿P==B?

- Dead node: (i==last) & (P+(S(i)==1) * A(i)) <> B)
- Live node: {¬ Dead node)} P+=(S(i) == 1) + A(i) ; {

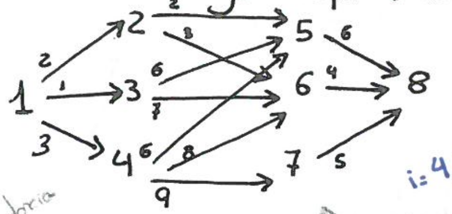
A(i)

↳(10——0) P=P+2= 0+2 = 2
(110—0) P=-3
(1——10) P=5
(1——1) D.N.
$(\underbrace{1——1}_{1^{er} subset}\ \overset{\downarrow}{2})$ D.N. En el 2º Subset

(1—— 13)
(1——20) ¿P?
(1—1211) P=3
(1—1212) P=2
(1 —1220) P=3
(1—1221)

- B.T Node: (j==3)} S(i) =0; i --; if (S(i) ==1) P -= A(i); {
- Solution node: (i == last)

## → Multistage Graph (Code)



i: 4,3

Distancia de → a →

| C: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | ... | | | | | | |
| 1 | 0 | 0 | 2 | 1 | 3 | 0 | 0 | 0 | |
| 2 | | | | | | | | | |
| 3 | | | | | | | | | |
| 4 | 0 | 0 | 0 | 0 | 0 | 6 | 8 | 9 | 0 |
| 5 | | | | | | | | | |
| 6 | | | | | | | | | |
| 7 | | | | | | | | | |
| 8 | | | | | | | | | |

DIRECTOS

| cost | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| | | 9 | 7 | 11 | 12 | 6 | 4 | 5 | 0 |

| d | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| | | 2 | 6 | 6 | 5 | 8 | 8 | 8 | 0 |

| Path | 0 | 1 | 2 | 3 | 4 | 5 | 6 | ... |
|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 6 | 8 | | | |

cost[i] = min}c[i][k] + cost[k]{

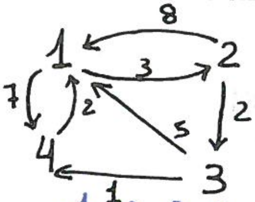
cost[4] = min} k=5  6+6=12
k=6  8+4=12
k=7  9+5=14
k=8 — (*)

cost[3] = min} k=4 — (*)
k=5  6+6=12
k=6  7+4=11
k=7 — (*)
k=8 — (*)

P[i] = d[P[i-1]]
P[2] = d[P[1]] = 2
P[3] = d[P[2]] = 6

```
main()}
   int stages, min;
   int n=8;
   int cost[9], d[9], path[9];
   intc[9][9]=}}...{,}...{,}...{...{;
  cost[n]=0;  8::0
  for(int i = n-1; i >= 1; i--)} -> 7 a 1
     min=32767; (número random)
     for(k=i+1; k<=n; k++)}
        if(c[i][k]!=0 && c[i][k]+c[k]<min)}
  Si no     min = c[i][k]+c[k];
  estén (*) d[i]=k;
  unidos   {
     {
   cost[i]=min;
 {
P   P[1]=1; P[stages]=n;
A
T   for(i=2; i<stages; i++)}
H     P[i]=d[P[i-1]];
```

## → All Pairs Shortest Path (Floyd-Warshall)



$$A^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & \infty \\ 3 & 5 & \infty & 0 & 1 \\ 4 & 2 & \infty & \infty & 0 \end{array}$$

$$A^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2^{(1)} & 15^{(2)} \\ 3 & 5 & 8^{(3)} & 0 & 1 \\ 4 & 2 & 5^{(4)} & \infty & 0 \end{array}$$

(1) $A^0[2,3]$ $A^0[2,1] + A^0[1,3]$
$2 < 8 + \infty$

(2) $A^0[2,4]$ $A^0[2,1] + A^0[1,4]$
$\infty > 8 + 7 = 15$

(3) $A^0[3,2]$ $A[3,1] + A^0[1,3]$
$\infty > 5 + 3 = 8$

(4) $A^0[4,2]$ $A^0[4,1] + A^0[1,2]$
$\infty > 2 + 3 = 5$

...

$$A^2 \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 5 & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array}$$

$A^1[1,3]$ $A^1[1,2] + A^1[2,3]$
$\infty > 3 + 2 = 5$

$A^1[1,4]$ $A^1[1,2] + A^1[2,4]$
$7 < 3 + 15$

$A^1[3,1]$ $A^1[3,2] + A^1[2,1]$
$5$ $8 + 8$

$A^1[4,3]$ $A^1[4,2] + A^1[2,3]$
$\infty > 5 + 2 = 7$

...

$$A^3 \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 5 & 6 \\ 2 & 7 & 0 & 2 & 3 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array}$$

$A^2[1,2]$ $A^2[1,3] + A^2[3,2]$
$3 < 5 + 8$

$A^2[1,4]$ $A^2[1,3] + A^2[3,4]$
$7 > 5 + 1 = 6$

$A^2[2,1]$ $A^2[2,3] + A^2[3,1]$
$8 > 2 + 5 = 7$

$$A^4 \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 5 & 6 \\ 2 & 5 & 0 & 2 & 3 \\ 3 & 3 & 6 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array}$$

...

$A^3[2,1]$ $A^3[2,4] + A^3[4,1]$
$7 > 3 + 2 = 5$

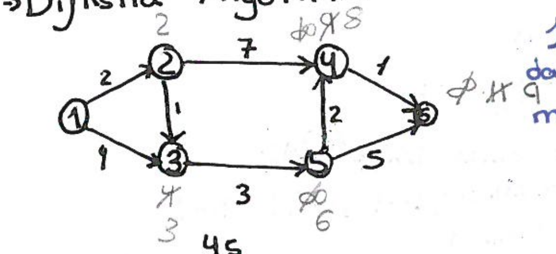
$A^3[3,1]$ $A^3[3,4] + A^3[4,1]$
$5$ $1 + 2 = 3$

$A^3[3,2]$ $A^3[3,4] + A^3[4,2]$
$8$ $1 + 5 = 6$

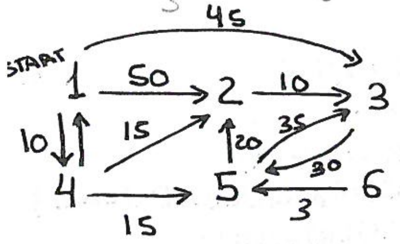
...

$$\tilde{A}^k[i,j] = \min\}A^{k-1}[i,j],\ A^{k-1}[i,k] + A^{k-1}[k,j]\{$$

# →Dijkstra Algorithm



1- Ir desde el menor actualizando datos y cantidades hasta llegar al menor posible



| Selected vertex | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| (Smallest) 4 | 50 | 45 | 10 | ∞ | ∞ |
| 5 | 50 | 45 | 10 | 25 | ∞ |
| 2 | 45 | 45 | 10 | 25 | ∞ |
| 3 | 45 | 45 | 10 | 25 | ∞ |
| 6 | 45 | 45 | 10 | 25 | ∞ |

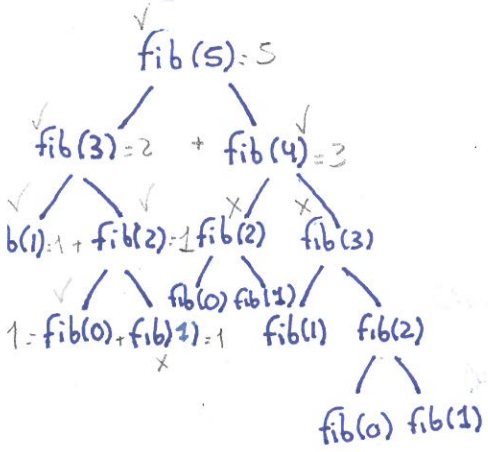
**Relaxation**

if (d[u] + c(u,v) < d[v])
  d[v] = d[u] + c(u,v)

# Dynamic Algorithms

Used to make codes more efficient

$$fib(n) = \begin{cases} 0 & n = 0 \\ 1 & n = 1 \\ fib(n-2) + fib(n-1) & n > 1 \end{cases}$$



```
int fib(int n)}
  if(n<=1)}
    return n;
  {
  return fib (n-2)+fib(n-1);
{
```

Con este código es
$\Theta(2^n)$
Se subdivide infinitamente

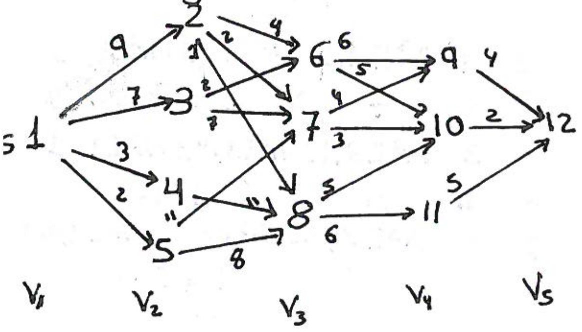
Fib

|  |  |  |  |  |  |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 2 | 3 | 5 |
| 0 | 1 | 2 | 3 | 4 | 5 |

i

```
int fib(int n)}
  if (n<= 1)}
  return n;
  F[0]=0;F[1]=1;{
  for (int i=2; i<=2; i++)}
   F[i]= F[i-2]+F[i-1];
  {
 return F[n];
{
```

# →Multistage Graph



| V | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cost | 16 | 7 | 9 | 18 | 15 | 7 | 5 | 7 | 4 | 2 | 5 | 0 |
| d | 2/3 | 7 | 6 | 8 | 8 | 10 | 10 | 10 | 12 | 12 | 12 | 12 |

$cost(i,j) = min\}c(j,l) + cost(i+1,l)\{$

$cost(\underset{i}{2},\underset{j}{3}) = min\}c(\underset{j}{3},\underset{l}{6}) + cost(\underset{i+1}{3},\underset{j}{6})$

d(1,1) = 2
d(2,2) = 7
d(3,7) = 10
d(4,10) = 12

d(1,1) = 3
d(2,3) = 6
d(3,6) = 10
d(4,10) = 12

BEST SOLUTIONS

$cost(\overset{stage}{x},\overset{vertex}{y})$

cost(3,6) = min}c(6,9) + cost(4,9), c(6,10) + cost(4,10){ = }6+4, 5+2{ = 7

cost(3,7) = min}c(7,9) + cost(4,9), c(7,10) + cost(4,10) = }4+4, 3+2{ = 5

cost(3,8) = min}c(8,10) + cost(4,10), c(8,11) + cost(4,11) = }5+2, 6+5{ = 7

cost(2,2) = min}c(2,6) + cost(3,6), c(2,7) + cost(3,7), c(2,8) + cost(3,8) = }11,7,8{ = 7

cost(2,3) = min}c(3,6) + cost(3,6), c(3,7) + cost(3,7){ = }2+7, 7+5{ = 9

cost(2,4) = min}c(4,8) + cost(3,8){ = 11 + 7 = 18

cost(2,5) = min}c(5,7) + cost(3,7) c(5,8) + cost(3,8) = }11+5, 8+7{ = 15

cost(1,1) = min}c(1,2) + cost(2,2), c(1,3) + cost(2,3), c(1,4) + cost(2,4), c(1,5) + cost(2,5)} =

=}9+7, 7+9, 3+18, 2+15{ =

# Trabajo → Greedy

MidElemento: $f + (s - f)/2$

↳ func(Array[22], $\overset{F}{\downarrow}$ 0, 22+1) ↙ size(s)

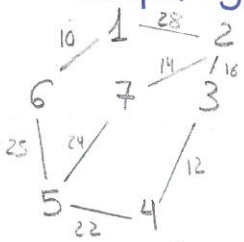
$m = 0 + (23-0)/2 = 11'5 \simeq 11$

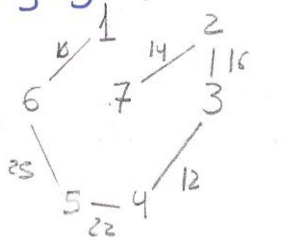
$$\left.\begin{array}{l} \text{Merge Sort (Array, 0, 11)} \longrightarrow n1 = m - f + 1 \\ \text{Mergesort (Array, 11+1, 22)} \longrightarrow n2 = s - m \end{array}\right\} \begin{array}{l} F[n1]. \\ S[n2] \end{array} \Big\{ \text{Merge, introduce from the lowest to highest}$$

# Greedy

- Ejercicios rondan con problemas matemáticas/programación
- Prims

Simple, coge el primero y sigue por el que menor cueste unido



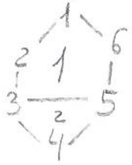


$\Theta(|V| \cdot |E|) = (n \times n) = (n^2)$

- Kruskals

Igual que Prims pero no hace falta seguir por uno unido. Evitar circulos

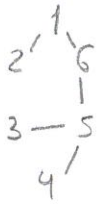
- Spanning trees



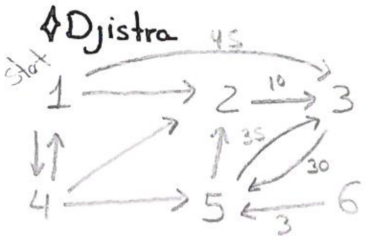
$|E| C_{|V-1|}$ – nº of cycles

$7 C_5 - 2 = 5$

⇒



- Djistra

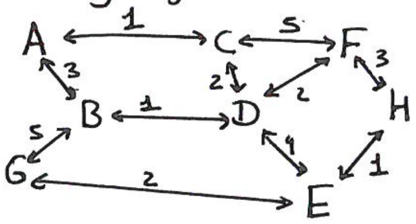


Ir desde el menor actualizando datos y cantidades hasta tocar todos

| Selected vertex | | | | | |
|---|---|---|---|---|---|
| 4 | 50 | 45 | ⑩ | ∞ | ∞ |
| 5 | 50 | 45 | ⑩ | ㉕ | ∞ |
| 2 | ㊺ | 45 | ⑩ | ㉕ | ∞ |
| 3 | ㊺ | ㊺ | ⑩ | ㉕ | ∞ |
| 6 | ㊺ | ㊺ | ⑩ | ㉕ | ∞ |

## AAD 2022 06 30

### Greedy Dijskstra



Optimal path: E, G, F, D, B, A
Total: 3+1+7+5+3+6+8: 33

| Selected Vertex | F | E | C | D | G | B | A |
|---|---|---|---|---|---|---|---|
| E | 3 | ① | ∞ | ∞ | ∞ | ∞ | ∞ |
| G | 3 | ① | ∞ | 4 | ③ | ∞ | ∞ |
| F | ③ | ① | 7 | ⑤ | ③ | 8 | ∞ |
| D | ③ | ① | ⑦ | ⑤ | ③ | ⑥ | ∞ |
| B | ③ | ① | ⑦ | ⑤ | ③ | ⑥ | ⑧ |
| A | | | | | | | |

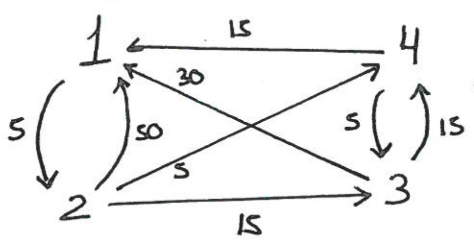
Candidates: Arcs / Edges
Order: Increasing order
Feasibility:

```
if(d[u] + c(u,v) < d[v]))
    d[v] = d[u] + c(u,v);
  { else }
    d[v] = d[v]
```

Selection: H first
Ending condition: All nodes checked

Ejercicio de la teoria

→ Floyd



$$A^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 5 & \infty & \infty \\ 2 & 50 & 0 & 15 & 5 \\ 3 & 30 & \infty & 0 & 15 \\ 4 & 15 & 15 & 5 & 0 \end{array}$$

Para formar la matriz $A_1$ debemos ir viendo los valores de la Matriz $A_0$, y así consecutivamente.
Paso por medio el 1, 2, 3 y 4.

$$A_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 5 & \infty & \infty \\ 2 & 50 & 0 & 15 & 5 \\ 3 & 30 & 35 & 0 & 15 \\ 4 & 15 & 15 & 5 & 0 \end{array}$$

$A^0[2,3] < A^0[2,1] + A^0[1,3]$
$15 \not< 50 + \infty$

$A^0[2,4] < A^0[2,1] + A^0[1,4]$
$5 \not< 50 + \infty$

$A^0[3,2] < A^0[3,1] + A^0[1,2]$
$\infty < 30 + 5 = 35$

$A^0[3,4] < A^0[3,1] + A^0[1,4]$
$15 \not< 30 + \infty$

$A^0[4,2] < A^0[4,1] + A^0[1,2]$
$15 \not< 15 + 5$

$A^0[4,3] < A^0[4,1] + A^0[1,3]$
$5 \not< 15 + \infty$

$$A_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 5 & 20 & 10 \\ 2 & 50 & 0 & 15 & 5 \\ 3 & 30 & 35 & 0 & 15 \\ 4 & 15 & 15 & 5 & 0 \end{array}$$

$A'[1,3] < A'[1,2] + A'[2,3]$
$\infty < 5 + 15 = 20$

$A'[1,4] < A'[1,2] + A'[2,4]$
$\infty < 5 + 5 = 10$

$A'[3,1] < A'[3,2] + A'[2,1]$
$30 \not< \infty + 50$

$A'[3,4] < A'[3,2] + A'[2,4]$
$15 \not< 35 + 5$

$A'[4,1] < A'[4,2] + A'[2,1]$
$15 \not< 15 + 50$

$A'[4,3] < A'[4,2] + A'[2,3]$
$5 \not< 15 + 15$

Las obvias no las trabajo ya

$A^2[2,1] < A^2[2,3] + A^2[3,1]$
$50 < 15 + 30 = 45$

$A^2[1,4] < A^2[1,3] + A^2[3,4]$
$10 \not< 20 + 15$

$A^2[4,1] < A^2[4,3] + A^2[3,1]$
$15 \not< 5 + 30$

$A^2[4,2] < A^2[4,3] + A^2[3,2]$
$15 \not< 5 + 35$

$$A_3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 5 & 20 & 10 \\ 2 & 45 & 0 & 15 & 5 \\ 3 & 30 & 35 & 0 & 15 \\ 4 & 15 & 15 & 5 & 0 \end{array}$$

SOLUCION

$$A_4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 5 & 15 & 10 \\ 2 & 20 & 0 & 10 & 5 \\ 3 & 30 & 30 & 0 & 15 \\ 4 & 15 & 15 & 5 & 0 \end{array}$$

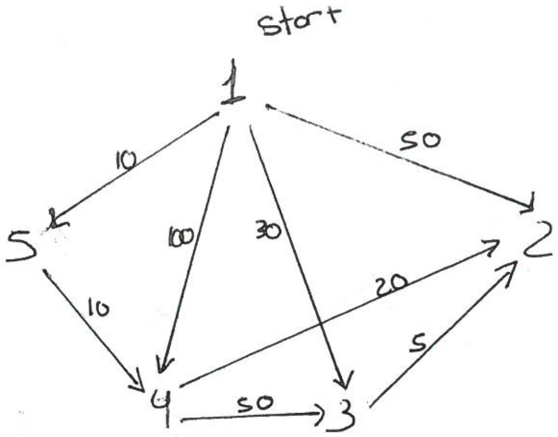
$A^3[1,3] < A^3[1,4] + A^3[4,3]$
$20 < 10 + 5 = 15$

$A^3[2,1] < A^3[2,4] + A^3[4,1]$
$45 < 5 + 15 = 20$

$A^3[2,3] < A^3[2,4] + A^3[4,3]$
$15 < 5 + 5 = 10$

$A^3[3,1] < A^3[3,4] + A^3[4,1]$
$30 \not< 15 + 15$

$A^3[3,2] < A^3[3,4] + A^3[4,2]$
$35 < 15 + 15 = 30$



Candidates: Nodes / arcs

Order: Increasing values

Feasibility: (d (u) + c(u,v) < d[v]) }

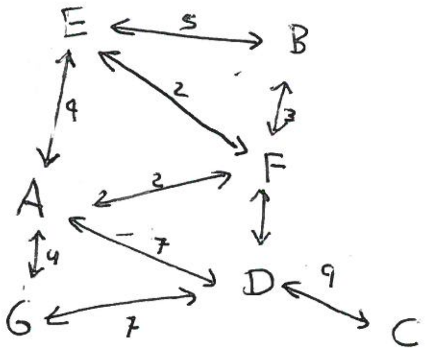
d[v] = d[u] + c(u,v);

{

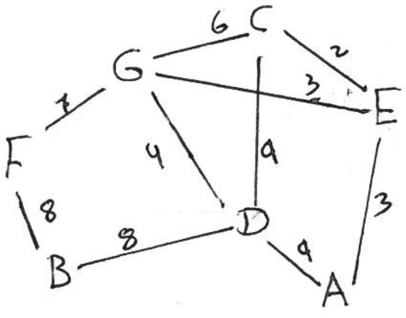
Finish: When we find the shortest path

| Vertex selected | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 5 | 50 | 30 | 100 | 10 |
| 4 | 50 | 30 | 20 | 10 |
| 3 | 75 | 70 | 20 | 10 |
| 2 | | | | |

40 + ∞

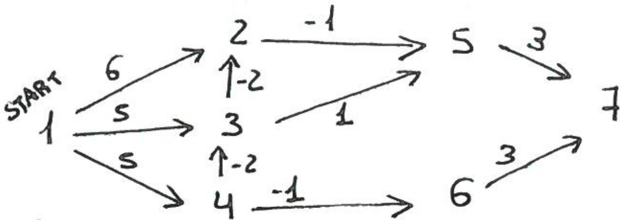


| vertex selected | E | G | B | F | D | C |
|---|---|---|---|---|---|---|
| F | 9 | 4 | ∞ | 2 | 7 | ∞ |
| E | 4 | 4 | 5 | 2 | 7 | ∞ |
| G | 9 | 4 | 5 | 2 | 7 | ∞ |
| B | 4 | 4 | 5 | 2 | 7 | ∞ |
| D | 4 | 4 | 5 | 2 | 7 | 16 |
| C | 4 | 4 | 5 | 2 | 7 | 16 |



| | E | D | C | G | B | F |
|---|---|---|---|---|---|---|
| E | 3 | 9 | ∞ | ∞ | ∞ | ∞ |
| C | 3 | 9 | 5 | 6 | ∞ | ∞ |
| G | 3 | 9 | 5 | 6 | ∞ | ∞ |
| D | 3 | 9 | 5 | 6 | ∞ | 13 |
| F | 3 | 9 | 5 | 6 | 17 | 13 |
| B | 3 | 9 | 5 | 6 | 17 | 13 |

## → Bellman-Ford



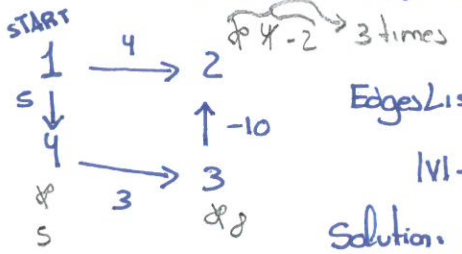
$\overset{\nearrow \text{vertex}}{|V|} = n = 7$

Max edges = |V|-1

= 7-1 = 6 times relaxation ↗ d[u] + c(u,v) < d[v]
d[v] = d[u] + c(u,v)

EdgesList: (1,2),(1,3),(1,4),(2,5),(3,2),(3,5),(4,3),(4,6),(5,7),(6,7)

Ejercicio + corto



EdgesList = (3,2),(4,3),(1,4),(1,2)

|V|-1 = 4-1 = 3

Solution. 1-0 ; 2- (-2) ; 3- 8 ; 4- 5

## 1/0 Knapsack Problem

m= 8
n= 4

P= }1,2,5,6{

w = }2,3,4,5{

Not divisible

$X = \}1,0,0\ldots\{ \rightarrow 2^4 = 2^n = \Theta$

$\max \sum p_i x_i$

$\sum w_i x \leq m$

| $P_i$ | $w_i$ | V | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 2 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 3 | 2 | 0 | 0 | 1 | 2 | 2 | 3 | 3 | 3 | 3 |
| 5 | 4 | 3 | 0 | 0 | 1 | 2 | 5 | 5 | 6 | 7 | 7 |
| 6 | 5 | 4 | 0* | 0 | 1 | 2 | 5 | 6 | 6 | 6 | |

| $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|
| 0 | 1 | 0 | 1 |

8-6=2
2-2=0

$V[\overset{\nearrow\text{row}}{i}, \overset{\nearrow\text{column}}{w}] = \max\}V[i-1,w], V[i-1, w-w[i]]+P[i]$

* $V[4,1] = \max\}V[3,1], \underbrace{V[3,1-5]+6}_{\text{undefined}}\{$

```
main () }
 int P[5]=}0,1,2,5,6{;
 int wt[5]=}0,2,3,4,5{;
 int m=8,n=4;
 int k[5][9]
for(int i=0;i<=n;i++)}
  for(int w=0;w<=m;w++)}
    if(i==0|| w==0)}
       k[i][w]=0;
    {elseif(wt[i]<=w)}
       k[i][w]= max(P[i]+k[i-1][w-wt[i]], k[i-1][w]);
    {else k[i][w]=k[i-1][w];
      {
   {
```

```
int Feb22(int n)}
  int i, j, k, m=0, p=0;
  if(n<10)} i=1;
          while(i<n)} for(j=1;j<=2^i; j++)} j=j+1
                    i++;{{
          else } for (i=1; i<=n; i++)} j=n/1;
                                for (k=1;k<j ; k++)} p++;{{
              while (m<=2)} p+= Feb22(n-2);
                                m++;{{

   return p;{
```

Size of problem = n

$$\text{Feb22} \begin{cases} \text{if } (n<10) \\ \\ \text{else} \end{cases}$$

$$\sum_{i=1}^{n} \left(3 + \sum_{j=1}^{2^i} 5\right) \Rightarrow \sum_{i=1}^{n} \left(3 + 5 \cdot 2^i\right)$$

$$3'n + 5 \sum_{i=1}^{n} 2^i ??$$

$\int x = \frac{x^2}{2}$

$\int \frac{1}{x} = \ln(x)$

```
1) int March21(int a, int b, int n)}
     int i, j, k, m, p = 0
     m = a/b;
     if(a<10b)}i=1;      *
               while(i<m)} for(j=1;j<=i;j++)}j=j+1;{
                                i++;{
               {
      else } for(i=1;i<=m;i++)} j=m/i    (*)
                                for(k=1;k<j;k++)} p++;{
No entra nunca [if(a==b)3*March21(a,3b,n/3);
          ⚠ else 5*March21(a/5,b,n/5);
                                  m/5
          {
     {
```

Cuando var sum
↳ var dentro

Size of the problem: m = a/b   pq el bucle depende de m

bucle dentro del while

March 21
- if (a<10b) → if(m<10) $\boxed{a/b<10}$ → $\sum_{i=1}^{m*}\left(3+\sum_{j=1}^{i}5\right) \Rightarrow \sum_{i=1}^{m}(3+5i)$ → $\int x = \frac{x^2}{2}$
- for O(m) → Θ(1)
- while(m²) → (0) → ✗
- else

$3m + 5\sum_{i=1}^{m} i$

$3m + \frac{5m^2}{2}$

Big Θ ≤ $m^2$

$\int \frac{1}{x} \cdot \ln x$

else → = j (*)

$\sum_{i=1}^{m}\left(5+\sum_{k=1}^{m/i}5\right) \to \sum_{i=1}^{m} 5 + \frac{5m}{i};\ 5m + 5m \cdot \sum_{i=1}^{m} 1/i \to \boxed{5m + 5m\,lg}$

3 casos Θ

$m/x \quad \sqrt[m]{x}$

$m-1$

for(k) Θ(m)

$\underbrace{\text{for}(i=1)\ \Theta(m \log m)}_{\text{Más grande (BigO)}}$

⚠ $m = 5^i$   Pq si

$t(m) = 5t(m/5) + k \cdot m \cdot \log(m)$

$t(5^i) - 5t(5^{i-1}) = k \cdot 5^i \cdot \log(5^i)$

↑ $\log_5$

$\underbrace{t(5^i) - 5t(5^{i-1})} = k \cdot 5^i \cdot i \to r = 5; \alpha = 1$

↓ $r = 1; \alpha = 1$

$5^i \cdot A$

$r^i - 5r^{i-1} = \frac{r^i}{r^{i-1}} - \frac{5\cancel{r^{i-1}}}{\cancel{r^{i-1}}};\ r^{(1-i+1)}$

$r - 5 = 0; r = 5$

$\alpha = 1$

3)2021

$$t(n)=\begin{cases} n+2 & (n<5) \\ t(\sqrt{n})+2t(\sqrt[4]{n})+\lg_2(n) & (o.w) \end{cases}$$

$x \; y \; 2 \to i$

$\lg \; \sqrt{\ }$

$t(n)=t(\sqrt{n})+2t(\sqrt[4]{n})+\lg_2(n)$

$t(n)-t(\sqrt{n})-2(\sqrt[4]{n})=\lg_2(n)$

$t(n)=t(2^{2^i})$ ; $t(2^{2^i})-t(2^{2^{i-1}})-2t(2^{2^{i-2}})=\lg_2(n)$

$t(2)=r$ ; $\frac{r^i}{r^{i-2}}-\frac{r^{i-1}}{r^{i-2}}\frac{-2r^{i-2}}{r^{i-2}}$ ; $r^2-r-2$

$\frac{1\pm\sqrt{1+4\cdot 2}}{2}=\frac{1\pm 3}{2}$ $\nearrow 2=r, \alpha=1$ $\searrow -1=r, \alpha=1$

$2^{2^i}$

$C2^i\cdot i$

$C \quad \lg_2(\lg_2(n))\cdot\lg_2(n)$

$t(n)=\lg_2(n)$

$t(2^{2^i})=\lg_2(2^{2^i})$

$2^i$

$r=2, \alpha=1$

$r=2, \alpha=2$

$r=(-1), \alpha=1$

$\lg_2(\lg_2(n))$

$t(2^{2^i})=A(-1)^i+B2^i+C2^i i$

$t(n)=A(-1)^i+B\log_2(n)+C\underbrace{(\log_2(n)\cdot\lg_2(n)}_{O}$

$\lg_2()$

$(\log_2(n)\cdot\lg_2(n)$

$i=0 \quad n=2$ $\{$ $t(2)=4$

$i=1 \quad n=4$ $\{$ $t(4)=6$

$i=2 \quad n=16$ $\{$ $t(16)=6+2\cdot 4+4=18$

$A+B=4$ ; $A=4-B$

$-\cancel{A}+2B+2C=6$

$A+4B+8C=18$

$-\cancel{4}+B+2B+\cancel{2}C=6$

$4-B+4B+8C=18$

$=$

$\cancel{3}B+2C=6$

$3B+8C=18$

$6C=12$

$\boxed{C=2>0}$

$\Theta=\lg_2(\lg_2(n))\lg_2(n)$

3)2020

$$t(n)=\begin{cases} n+2 & (n<3) \\ 16t(\sqrt[4]{n})+\lg_2(n) & (o.w) \end{cases}$$

$\sqrt{\ }$ $x \; y \; 2 \to i$ $\log$

$t(n)=16t(\sqrt[4]{n})+\lg_2(n)$

$t(n)-16t(\sqrt[4]{n})-\lg_2(n)$

$t(n)=t(2^{4^i})$ ; $t(2^{4^i})-16t(2^{4^{i-1}})=\lg_2(2^{4^i})$

$f(2^4)=r$ ; $r^i-16r^{i-1}$

$r=16, \alpha=1$

$\lg_2(2^{4^i})$ ; $4^i$

$r=4, \alpha=1$

$t(2^{4^i})=A\cdot 16^i+B4^i$

$t(n)=\underbrace{A\cdot\lg_2(n)^2}_{O(\lg_2(n))^2}+B\cdot\lg(n)$

$i=0 \quad n=2 \qquad t(2)=4$

$i=1 \quad n=16 \qquad t(16)=16\cdot 4+$

$2)^{2018}\quad f(n) = \begin{cases} n & (n<5) \\ 5f(n/2)-4f(n/4)+2n & (o.w) \end{cases}$

$f(n) = 5f(n/2) - 4f(n/4) + 2n \qquad \frac{n}{x};\ n:x^i$

$f(n) - 5f(n/2) + 4f(n/2^2) = 2n$

$f(2^i) - 5f(2^i/2) + 4f(2^i/2^2) = 2n$

$\tilde{f}(2) = \tilde{r} \qquad r^i - 5r^{i-1} + 4r^{i-2} = 2n$

$\frac{r^i}{r^{i-2}} - 5\frac{r^{i-1}}{r^{i-2}} + 4\frac{r^{i-2}}{r^{i-2}} = 2n$

$r^2 - 5r + 4 = 2n$

$\frac{5 \pm \sqrt{25 - 4\cdot 4\cdot 1}}{2}$ ↗ $4 \quad r=4,\ \alpha:1$ ↘ $1 \quad r=1,\ \alpha:1$

$f(n) = 2n$

$f(2^i) = \underbrace{2\cdot 2^i}_{r=2,\ \alpha=1}$

$f(2^i)\ A + B2^i + C4^i$

$\boxed{\begin{array}{l} r=4,\ \alpha=1 \\ r=2,\ \alpha=1 \\ r=1,\ \alpha=1 \end{array}}$

$f(n) = A + Bn + Cn^2$

$O(n^2),\ \Omega(n^2),\{\ \Theta(n^2)$

$2)^{2017}\quad f(n) = \begin{cases} n^2 & (n<6) \\ 4f(n/5)-3f(n/25)+n+lg_5(n) & (o.w) \end{cases}$

$f(n) = 4f(n/5) - 3f(n/25) + n + lg_5(n) \qquad \frac{n}{x};\ n:x^i$

$f(n) - 4f(n/5) + 3f(n/5^2) = n + lg_5(n)$

$f(5^i) - 4f(5^i/5) + 3f(5^i/5^2) = n + lg_5(n)$

$f(5^i) - 4f(5^{i-1}) + 3f(5^{i-2})$

$\tilde{f}(5) = \tilde{r} \qquad r^i - 4f\,r^{i-1} + 3f\,r^{i-2}$

$\frac{r^i}{r^{i-2}} - 4\frac{r^{i-1}}{r^{i-2}} + 3\frac{r^{i-2}}{r^{i-2}};\ r^2 - 4r + 3$

$\frac{4 \pm \sqrt{16 - 4\cdot 3\cdot 1}}{2}$ ↗ $r=3,\ \alpha:1$ ↘ $r=1,\ \alpha:1$

$n + lg_5(n) = f(n)$

$\underbrace{5^i}_{r=5,\ \alpha:1} + lg_5(5^i) = f(5^i)$

$r=1$, $\alpha=2$

$r=1,\ \alpha=3$

$r=3,\ \alpha=1$

$r=5,\ \alpha=1$

$f(5^i) : A + Bi + Ci^2 + D3^i + E5^i$

$f(n) : A + B\,lg_5(n) + C\,lg_5(n))^2 + Dn^{lg_5(n)} + 5n$

$O(n),\ \Omega(n)\{\ \Theta(n)$

$2)^{2015}\quad f(n) = \begin{cases} 0 & (n=0) \\ 3f(n-1)+2+2^n & (o.w) \end{cases}$

$f(n) = 3f(n-1) + 2 + 2^n$

$f(n) - 3f(n-1) = 2 + 2^n$

$f(n) = \tilde{r};\ r^n - 3r^{n-1} = 2 + 2^n$

$\frac{r^{\cancel{n}}}{\cancel{n-1}} - 3\frac{\cancel{r^{n-1}}}{\cancel{r^{n-1}}};\ r-3;\ r=3,\ \alpha=1$

$\underbrace{1\cdot 2}_{r=1,\ \alpha=1} + \underbrace{2^n}_{r=2,\ \alpha=1}$

$r=1,\ \alpha=1$

$r=2,\ \alpha=1$

$r=3,\ \alpha=1$

$A + B2^n + C3^n$

$O(3^n),\ \Omega(3^n)\{\ \Theta(3^n)$

$3)\ 2015\quad f(n) = \begin{cases} n & (n<6) \\ 2f(2n/5)+5n & (o.w) \end{cases} \qquad \frac{n}{x};\ n:x^i$

$f(n) = 2f(n/5/2) + 5n$

$-(\frac{5}{2})^i = 2f(5/2^i/5/2) = 5n$

$(\frac{5}{2})^i = 2f(5/2^{i-1}) = 5n$

$(5/2)^i = \tilde{r} \qquad r^i - 2r^{i-1}$

$\frac{-i}{i-1} - 2\frac{\cancel{r^{i-1}}}{\cancel{r^{i-1}}};\ r=2,\ \alpha=1$

$f(n) = 5n$

$f(\frac{5}{2})^i = \underbrace{5\cdot(\frac{5}{2})^i}_{r=5/2,\ \alpha=1}$

$A2^i + B\cdot(\frac{5}{2})^i$

$f(n) : An^{0'8} + Bn$

$O(n),\ \Omega(n)\ (\ \Theta(n)$

2)$^{2023}$ $f(n)=\begin{cases} 3n & (n<4) \\ 3f(n-1)-2f(n-2)+n+3^n & (o.w) \end{cases}$

$f(n)=3f(n-1)-2f(n-2)+n+3^n$

$f(n)-3f(n-1)+2f(n-2)=n+3^n$

$f=r$ $\quad r^n-3r^{n-1}+2r^{n-2}=n+3n$

$\frac{r^n}{r^{n-2}}-3\frac{r(n-1)}{r(n-2)}+2\frac{r(n-2)}{r(n-2)}=n+3n$

$r^2-3r+2=n+3^n$

$\frac{3\pm\sqrt{9-4\cdot1\cdot2}}{2}$ $\to 2;\ r=2,\ \alpha=1$ ; $\to 1\quad r=1,\ \alpha=1$

$=0$

$\underbrace{1n}+\underbrace{3^n}_{r=3;\alpha=1}$

$1^n\cdot(n)$
$5^n\cdot(p)$
~~$r=1\ \alpha=2$~~

$r=3,\ \alpha=1$
$r=2,\ \alpha=1$
$r=1,\ \alpha=3$

$f(n)=$ $A+Bn+Cn^2+D2^n+E3^n$

$O(3^n)\quad\Omega(3^n)\}\ \Theta(3^n)$

2)$^{2020}$ $f(n)=\begin{cases} n^2 & (n<5) \\ 5f(n-1)-8f(n-2)+4f(n-3)+5n2^n+3^n & (o.w) \end{cases}$

$f(n)-5f(n-1)+8f(n-2)-4f(n-3)=5n2^n+3^n$

$f(n)=r^n$ $\quad r^n-5r^{(n-1)}+8r^{n-2}-4r^{n-3}=5n2^n+3^n$

$\frac{r^n}{r^{n-3}}-5\frac{r^{n-1}}{r^{n-3}}+8\frac{r^{n-2}}{r^{n-3}}-4\frac{r^{n-3}}{r^{n-3}}=5n2^n+3^n$

$r^3-5r^2+8r-4=5n2^n+3^n$

$$\begin{array}{c|cccc} & 1 & -5 & 8 & -4 \\ ① & & 1 & -4 & 4 \\ \hline & 1 & -4 & 4 & 0 \end{array}$$

$x^2-4x+4$

$\frac{4\pm\sqrt{16-4\cdot1\cdot4}}{2}\to 2$ por 1 solo

$r=1,\ \alpha=1$
$r=2,\ \alpha=2$

$\underbrace{5n2^n}_{r=2,\alpha=1+1}+3^n$ $r=3,\ \alpha=1$

$r=3,\ \alpha=1$
$r=2,\ \alpha=4$
$r=1,\ \alpha=1$

$f(n)=A+(B+Cn+Dn^2+En^3)2^n+F3^n$

$f(n)=A+B2^n+Cn2^n+Dn^2\cdot2^n+En^3\cdot2^n+F3^n$

$O(3^n),\ \Omega(3^n)\}\ \Theta(3^n)$

2)$^{2019}$ $f(n)=\begin{cases} 3n^2 & (n<5) \\ 4f(n/2)-3f(n/4)+5n+3n^2 & (o.w) \end{cases}$

$\frac{n}{x};\ n=x^i$ $\quad f(n)=4f(n/2)-3f(\overset{2^i}{n/4})+5n+3n^2$

$f(n)-4f(n/2)+3f(n/2^i)=5n+3n^2$

$f(2^i)-4f(2^i/2)+3f(2^i/2^2)=5n+3n^2$

$f(2^i)-4f(2^{i-1})+3f(2^{i-2})=5n+3n^2$

$f(2)=r$ $\quad r^i-4r^{i-1}+3r^{i-2}=5n+3n^2$

$\frac{r^i}{r^{i-2}}-4\frac{r^{i-1}}{r^{i-2}}+3\frac{r^{i-2}}{r^{i-2}}=5n+3n^2$

$r^2-4r+3=5n+3n^2$

$\frac{4\pm\sqrt{16-4\cdot1\cdot3}}{2}=$ $3\ r=3,\ \alpha=1$ ; $1\ r=1,\ \alpha=1$

$f(n)=5n+3n^2$

$f(2^i)=\underbrace{5\cdot2^i}_{r=2,\alpha=1}+\underbrace{3\cdot4^i}_{r=4,\alpha=1}$

"¿A cuánto hay que elevar n para que de T(2)?"

$r=1,\ \alpha=1$
$r=2,\ \alpha=1$
$r=3,\ \alpha=1$
$r=4,\ \alpha=1$

$f(n)=A+Bn+Cn^{\lg_2(3)}+Dn^2$

$O(n^2),\ \Omega(n^2)\}\ \Theta(n^2)$

3/2019

⚠ $$t(n) = \begin{cases} n+2 & (n<511) \\ 3t(\sqrt[3]{n}) + 4t(\sqrt[9]{n}) + \log_2(n) \end{cases}$$

$\log_n \leftarrow x \quad y \quad z \quad 2^{3^i}$

$$t(2^{3^i}) = 3t(2^{3^{i-1}}) + 4t(2^{3^{i-2}}) + \log_2(2^{3^i})$$

(*) $$t(2^{3^i}) - 3t(2^{3^{i-1}}) - 4t(2^{3^{i-2}}) = 3^i$$

$$r^i - 3r^{i-1} - 4r^{i-2} = 3^i$$

$$r^2 - 3r - 4 = 3^i$$

$$\frac{3 \pm \sqrt{9-4\cdot4\cdot1}}{2} \begin{cases} \nearrow 4 \\ \searrow -1 \end{cases}$$

$i=0 \quad n=2$ (*)
$i=1 \quad n=8$
$i=2 \quad n=512$

⚠ $t(2) = 4$
$t(8) = 10$
$t(512) = 3\cdot10 + 4\cdot4 + 9 = 55$

$r=3, \alpha=1$
$r=4, \alpha=1$
$r=(-1), \alpha=1$

$t(3^{2^i}) = A\,3^i + 4^i B + C(-1)^i$

$t(n) = \log_2(n) A + \log_2(n)^{\log_3(4)} B + (-1)^i \cdot C$

$t(2) = 4 = A + B + C$
$t(8) = 10 = 3A + 4B - C$
$t(512) = 55 = 9A + 16B + C$

3/2023

$$t(n) = \begin{cases} n^2 & (n<5) \\ 6t(\sqrt{n}) - 8t(\sqrt[4]{n}) + 16 \end{cases} \quad x\ y^2$$

$$t(n) = 6t(\sqrt{n}) - 8t(\sqrt[4]{n}) + 16$$

$$t(2^{2^i}) - 6t(2^{2^{i-1}}) + 8t(2^{2^{i-2}}) = 16$$

$r = t(2)$ ; $\frac{r^i}{r^{i-2}} - \frac{6r^{i-1}}{r^{i-2}} + \frac{8r^{i-2}}{r^{i-2}}$

$r^2 - 6r + 8$

$$\frac{6 \pm \sqrt{36 - 4\cdot8\cdot1}}{2} = \frac{6\pm2}{2} \begin{cases} \nearrow 4 = r, \alpha = 1 \\ \searrow 2 = r, \alpha = 1 \end{cases}$$

$r = 1, \alpha = 1$

$t(2^{2^i}) = A + B2^i + C4^i$

$t(n) = A + B\log_2(n) + \underbrace{C(\log_2(n))^2}_{O(\log_2(n))^2}$

$i=0 \quad n=2$
$i=1 \quad n=4$
$i=2 \quad n=16$

$t(2) = 4$
$t(4) = 16$
$t(16) = 6\cdot16 - 8\cdot4 + 16 = 80$

$t(2) = 4 = A + B + C$
$t(4) = 16 = A + 2B + 2C$
$t(16) = 80 = A + 4B + 16C$

$C = \boxed{10/3} > 0$

$\Rightarrow \Theta(\log(n))^2$

2) 2016

$$t(n) = \begin{cases} n & (n<3) \\ 4t(n-1) - 3t(n-2) + 3^n \end{cases}$$

$t(n) = 4t(n-1) - 3t(n-2) + 3^n$

$t(n) - 4t(n-1) + 3t(n-2) = 3^n$

$\tilde{r} = \tilde{t(n)}$; $r^n - 4r^{n-1} + 3r^{n-2} = 3^n$

$\frac{r^n}{r^{n-2}} - 4\frac{r^{n-1}}{r^{n-2}} + 3\frac{\cancel{r^{n-2}}}{\cancel{r^{n-2}}} = 3^n$

$r^2 - 4r + 3 = 3^n$

$\frac{4 \pm \sqrt{16 - 4 \cdot 3 \cdot 1}}{2}$ $\nearrow r = 3, \alpha = 1$ $\searrow r = 1, \alpha = 1$

$3^n \Rightarrow r = 3, \alpha = 1$

$r = 3, \alpha = 2$

$r = 1, \alpha = 1$

$A \quad + B3^n \quad + C\,3^n n$

$O(n \cdot 3^n), \Omega(n \cdot 3^n) \} \; \Theta(n \cdot 3^n)$

2) 2014

$$t(n) = \begin{cases} n & (n<3) \\ 9t(n/3) - 8t(n/9) + 2n^2 + 6n & (o.w) \end{cases}$$

$t(n) = 9t(n/3) - 8t(n/3^2) + 2n^2 + 6n$ $\quad \tilde{\frac{n}{x}}; \tilde{n = x^i}$

$t(n) - 9t(n/3) + 8t(n/3^2) = 2n^2 + 6n$

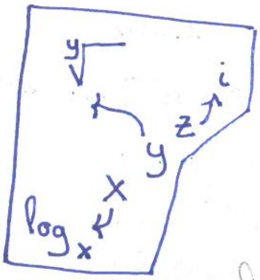
$t(3^i) - 9t(3^i/3) + 8t(3^i/3^2) = 2n^2 + 6n$

$t(3^i) - 9t(3^{i-1}) + 8t(3^{i-2})$

$\tilde{t(3)} = \tilde{r}$; $r^i - 9r^{i-1} + 8r^{i-2}$

$\frac{r^i}{r^{i-2}} - 9\frac{r^{i-1}}{r^{i-2}} + 8\frac{\cancel{r^{i-2}}}{\cancel{r^{i-2}}}$; $r^2 - 9r + 8 = 2n^2 + 6n$

$\frac{9 \pm \sqrt{81 - 4 \cdot 8 \cdot 1}}{2} = \frac{9 \pm 7}{2}$ $\nearrow 1 = r, \alpha = 1$ $\searrow r = 8, \alpha = 1$



$\log_2(n) = 2^{3^i}$

3) 2022

$$t(n) = \begin{cases} n/2 & (n<9) \\ 8t(\sqrt[3]{n}) - 15t(\sqrt[9]{n}) \end{cases}$$

$t(2^{3^i}) = 8t(2^{3^{i-1}}) - 15t(2^{3^{i-2}})$

$r^i = 8r^{i-1} - 15r^{i-2}$

$\frac{r^i}{r^{i-2}} - \frac{8r^{i-1}}{r^{i-2}} + 15\frac{r^{i-2}}{r^{i-2}} = 0$

$r^2 - 8r + 15 = 0$

$\frac{8 \pm \sqrt{64 - 4 \cdot 15 \cdot 1}}{2}$ $\nearrow 5$ $\searrow 3$

$r = 5, \alpha = 1$ $\quad t(2^{3^i}) = A5^i + B3^i$

$r = 3, \alpha = 1$ $\quad t(n) = \underbrace{\log_2(n)^{\log_3(5)}}_{O} A + \log_2(n) \cdot B$

$2^{3^i}$

$i = 0 \quad n = 2$

$i = 1 \quad n = 8$

$i = 2 \quad n = 512$

$n = 2 \quad t(2) = 1$

$n = 8 \quad t(8) = 4$ $\quad n/2$

$n = 512 \quad t(512) = 8t(2^3) - 15t(2) = 8 \cdot 4 - 15 = 17$

$t(8) = 4 = 5A + 3B$ $\quad \cdot 5 \quad 20 = 25A + 15B$

$t(512) = 17 = 25A + 9B$ $\quad 17 = 25A + 9B$

$4 = 5A + 9/6$ $\quad 3 = 6B$; $\boxed{B = 3/9}$

$\boxed{A = \frac{15/6}{5}} > 0$ ✓✓

Significa que $O(\log_2(n)^{\log_3(5)}) = \Theta(\log_2(n)^{\log_3(5)})$

```
int March2019 (int a, int b, int c)}
    int  m:b-a;
    int  A: a;
    int B: b;
    int k, t;
    while (a≤b)} k: 1; t: 1;
        for (i: 1, m, i++)} t: t+m; {
        while (k<t)} k: k+2;
                  *for (i: 1, k, i++)} c++; {
                  {
        do} k: k /3; { while (k >0);
         a++; b--{
     {
```

3 questions b-a

Done m times
$t=m^2$??

**Size of problem** $m=b-a;$

while(k<t)  for(i=1)  outer for

$$\sum_{l=1}^{m/2}\left(8+\sum_{j=1}^{m^2/2}\left(\sum_{i=1}^{2j}5\right)+\sum_{i=1}^{m}5+5\lg_3(m^2)\right)$$

$2j=k$

$$\underbrace{t(m)-2t(m-2)}=\underbrace{km^5}$$

$r=\sqrt{2}$ $\alpha=1$ $r=1$ $\alpha=6$

$r=\sqrt{2}$ $\alpha=1$ $\Downarrow$ $O(m^5)$

nk. k!>x

```
int Feb22(int n)}
    int i, j, k, m: 0, p: 0;
    if (n<10)} i: 1
        while (i<n)} for (j=1; j<= 2^i; j++)} j=j+1; { ∑
                  i++; {{
    else } for (i: 1; i<= n; i++)} j=n/i;
                                  for (k: 1; k<j; k++)} p++; {{ ∑
        while (m≤ 2)} p+= Feb22(n-2);
                      m++; {{
    return p; {
```

$2^n$, at most $2^{10}$ (1024)

3 times, $\pm\sqrt{3}$

(*)

**Size of problem:** n

$$T(n)=n\left(T(n/2)\right)^2;\ T(1)=1/3 \Rightarrow \boxed{\begin{array}{l}T(1)=3^{-1}\\ T(2)=2\cdot 3^{-2}\\ T(4)=4(2\cdot 3^{-2})^2=2^4\cdot 3^{-4}\end{array}}$$

$$xT(2^i)-2^i(T(2^{i-1}))^2=0$$

$$xr^i-2^i(r^{i-1})^2=0$$

$$\rightarrow \lg_2(T(n))=\boxed{\lg_2(n\cdot(T(n/2))^2)=\lg_2(n)+2\lg_2(T(n/2))}\Rightarrow n=2^i$$

$r=2$ $\alpha=1$

$$\Rightarrow \underbrace{\lg_2(T(2^i))}_{r^i}\underbrace{-2\lg_2(T(2^{i-1}))}_{-2r^{i-1}}=\textcircled{i}\quad r=1\ \alpha=2$$

$$\text{Sol}: T(n)=\frac{(4/3)^n}{4n}$$

$k$ (constant)

$n=2^i$

$$\lg_2(T(2^i))=A+Bi+C\cdot 2^i$$

$$2^{\lg_2(T(2^i))}=2^{(A+Bi+C2^i)};\ T(2^i)=2^A\cdot(2^i)^B\cdot(2^C)^{2^i}\Rightarrow T(n)=k_1\cdot n^{k_2}\cdot k_3^{\,n}$$

$$(*)\ 3^{-1}=k_1k_3 \nearrow k_1=3^{-1}\cdot k_3^{-1}\longrightarrow\boxed{k_1=2^{-2}}$$

$$2\cdot 3^{-2}=k_1\cdot 2^{k_2}k_3^2\Rightarrow 2\cdot 3^{-1}=2^{k_2}k_3\rightarrow 2^{1-k_2}\cdot 3^{-1}=k_3\rightarrow\boxed{2^2\cdot 3^{-1}=k_3}$$

$$2^4\cdot 3^{-4}=k_1\cdot 2^{2k_2}\cdot k_3^4\Rightarrow 2^4\cdot 3^{-3}=2^{2k_2}\cdot 2^{3-3k_2}\cdot 3^{-3}\Rightarrow 2^4=2^{3-k_2}\Longrightarrow\boxed{k_2=-1}$$

2021

2) $$t(n) = \begin{cases} n^2 & (n<5) \\ 5t(n/4) - 6t(n/16) + 2n + 2^2 n \end{cases}$$

$t(n) = 5t(n/4) - 6t(\overbrace{n/16}^{4^i}) + 2n + 2^2 n$; $n/x$; $n = x^i$

$t(4^i) - 5t(4^i/4) + 6t(4^i/4^2) = 2 \cdot 4^i + 2^2 \cdot 4^i$

$t(4^i) - 5t(4^{i-1}) + 6t(4^{i-2}) = 2 \cdot 4^i + 2^2 \cdot 4^i$

$t(4) = r$ $\quad r^i - 5r^{i-1} + 6r^{i-2} = 2 \cdot 4^i + 2^2 \cdot 4^i$

$2 \cdot 4^i + 2^2 \cdot 4^i$

$O = \overset{o}{} \Leftarrow 6 \cdot 4^i$ $\quad b^n \cdot (p) \Rightarrow \alpha + 1$; $b \Rightarrow r$

$r = 4; \alpha = 1$

$\frac{r^i}{r^{i-2}} - \frac{5r^{i-1}}{r^{i-2}} + \frac{6r^{i-2}}{r^{i-2}} = 2 \cdot 4^i + 2^2 \cdot 4^i$

$r^2 - 5r + 6 = 2 \cdot 4^i + 2^2 \cdot 4^i$

$\frac{+5 \pm \sqrt{25 - 4 \cdot 1 \cdot 6}}{2}$ → 3 $\quad r = 3; \alpha = 1$; → 2 $\quad r = 2; \alpha = 1$

$\Rightarrow \overset{o}{} = 0$

$0 + 1 = \alpha$

$r = 4, \alpha = 1$

$r = 3, \alpha = 1$

$r = 2, \alpha = 1$

$t(4^i) = 4^i \cdot A + 3^i \cdot B + 2^i \cdot C$

$t(n) = n \cdot A + 4^{i \log_4(3)} \cdot B + 4^{i \log_4(2)}$

$\left.\begin{matrix} O(n) \\ \Omega(n) \end{matrix}\right\} \Theta(n)$

2022

2) $$t(n) = \begin{cases} n^2 & (n<4) \\ 5t(n/3) - 6t(n/9) + 2n \lg_3(n) + 4n^2 & (o.w) \end{cases}$$

$t(n) = 5t(n/3) - 6t(\overbrace{n/9}^{3^2}) + 2n \lg_3(n) + 4n^2$ $\quad n/x$; $n = x^i$

$t(n) - 5t(n/3) + 6t(n/3^2) = 2n \lg_3(n) + 4n^2$

$t(3^i) - 5t(3^i/3) + 6t(3^i/3^2) = 2n \lg_3(n) + 4n^2$

$t(3^i) - 5t(3^{i-1}) + 6t(3^{i-2}) = 2n \lg_3(n) + 4n^2$

$t(3) = r$ $\quad r^i - 5r^{i-1} + 6r^{i-2} = 2n \lg_3(n) + 4n^2$

$\frac{r^i}{r^{i-2}} - 5\frac{r^{i-1}}{r^{i-2}} + 6\frac{r^{i-2}}{r^{i-2}} = 2n \lg_3(n) + 4n^2$

$r^2 - 5r + 6 = 2n \lg_3(n) + 4n^2$

$\frac{5 \pm \sqrt{25 - 4 \cdot 1 \cdot 6}}{2}$ → 3; $\underbrace{r = 3, \alpha = 1}$ → 2; $r = 2, \alpha = 1$ →

$2 \cdot 3^i \cdot \lg_3(3^i) + 4 \cdot (3^i)^2$

$2 \cdot \underbrace{3^i}_{} \cdot i + 4 \cdot \underbrace{9^i}_{}$

$\underbrace{r = 3 \; \alpha = 2}$ $\quad r = 9 \quad \alpha = 1$

$$\boxed{\begin{matrix} r=2 & \alpha=1 \\ r=3 & \alpha=3 \\ r=9 & \alpha=1 \end{matrix}}$$

de sumar $\alpha = 2 + \alpha = 1$

$t(3^i) = 2^i \cdot A + 3^i \cdot B + 3^i C i + 3^i \cdot D i^2 + 9^i E$

$t(n) = A n^{\lg_3(2)} + Bn + Cn \lg_3(n) + Dn(\lg_3(n))^2 + En^2$

$\Omega(n^2), O(n^2) \{ \Theta(n^2)$

```
Int Feb23 (int x, int y)}
    int m= x-y;
    int n= x/y;
    int i, j. k, t;
    while (x>y) }k=1; t=0;
                for (i=1, n, i++)} t= t+n;{
                while(k²<t)}k++;
                            for (j=1, n/k, j++)}t++;{
                            {
            y=3y;
            {
    m= 3 Feb23 (n,3) - Feb (n/3, 1);
     return t+m;
  {
```

$\Sigma$

Size of problem; $n = x/y$;

Feb23 $\left\{ \text{while} \quad \sum_{i=1}^{n} (3 + \Sigma \right.$

2) $t(n) = \begin{cases} n^2 & (n<5) \\ 5t(n-1) - 8t(n-2) + 4t(n-3) + 5n2^n + 3^n & (o.w) \end{cases}$

$t(n) = 5t(n-1) - 8t(n-2) + 4t(n-3) + 5n2^n + 3^n$

$t(n) - 5t(n-1) + 8t(n-2) - 4t(n-3) = 5n2^n + 3^n$

$r = t(n)^2 \quad r^n - 5r^{n-1} + 8r^{n-2} - 4r^{n-3}$

$r^3 - 5r^2 + 8r - 4$

$$\begin{array}{c|cccc} & 1 & -5 & 8 & -4 \\ 1 & & 1 & -4 & 4 \\ \hline & 1 & -4 & 4 & 0 \end{array}$$

$r=1, \alpha=1$

$\frac{4 \pm \sqrt{16 - 4 \cdot 4 \cdot 1}}{2} = r=2, \alpha=2$

$t(n) = 5n2^n + 3^n$

$r=2, \alpha=1+1 \qquad r=3, \alpha=1$

$r=1, \alpha=1$

$r=2, \alpha=4$

$r=3, \alpha=1$

$A + B2^n + C2^n i + D2^n i^2 + E2^n i^3 \underbrace{F3^n}$

$\left.\begin{array}{l} O(3^n) \\ \Omega(3^n) \end{array}\right\} \Theta(3^n)$

3) $t(n) = \begin{cases} n+2 & (n<3) \\ 16t(\sqrt[4]{n}) + lg_2 n & (o.w) \end{cases} \quad x \; y^2 \quad 2^{4^i}$

$t(n) = 16t(\sqrt[4]{n}) + lg_2 n$

$t(n) - 16t(\sqrt[4]{n}) = lg_2 n$

$t(2^{4^i}) - 16t(2^{4^{i-1}}) = 4^i$

$r^i - 16 \qquad \downarrow$

$r = 16, \alpha = 1 \qquad r=4, \alpha=1$

$t(2^{4^i}) = A16^i + B4^i$

$t(n) = A(\log_2 n)^2 + B \log_2 n$

$O(\log_2(n))^2$

$i=0 \quad n=2$

$i=1 \quad n=16$

$t(2) = 4$

$t(16) = 64 + 4 = 68$

$A + B = 4$

$16A + 4B = 68$

$A = 4 - B$

$16(4-B) + 4B = 68$

$64 - 16B + 4B = 68;$

$-12B = 4$

$A = 4 - (-1/3)$

$\boxed{A > 0}$

$\hookrightarrow \Theta((\log(n))^2)$

$B = -1/3$

$T(n) = 4T(n-1) - 2^n$ When $n > 0$ $T(0) = 1$

$$\underbrace{T(n) - 4T(n-1)}_{r=4 \quad \alpha:1} = \underbrace{-2^n}_{r:2;\ \alpha:1}$$

$T(0) = 1 \rightarrow 1 = A + B$

$T(1) = 2 \rightarrow 2 = A \cdot 2 + B4$

$0 = 2B \Rightarrow B = 0$

$1 = A$

$T(n) = A \cdot 2^n + B \cdot 4^n \quad \in O(4^n)$

1/

```
      int Feb23(int x, int y)}
          int m = x-y;
          int n = x/y;
          int i,j,k,t;
lg3(n) * while(x>y)} k=1; t=0;
  O(n) n <- for (i=1,n,i++)} t=t+n;{
 *O(n lg(n)){ n <- while(k^2<t)}k++;
      O(n) <= n <- for(j=1, n/k,j++)} t++;{
O(n lg^2(n))  y=3y;
          {
      m = 3 Feb23 (n,3) - Feb23 (n/3,1);
      return t+m;
      {
```

Size of problem = $n = x/y$

$$\sum_{k=1}^{n}\left(3 + \sum_{j=1}^{n/k} 5\right)$$

$$\Uparrow$$

$$\sum 4 + \left(\sum_{i=1}^{n} 5 + \boxed{\sum_{k=1}^{n}\left(3 + \sum_{j=1}^{n/k} 5\right)}\right) \quad \int 1/k =$$

$$\hookrightarrow \sum_{k=1}^{n}\left(3 + 5\frac{n}{k}\right) = 3n + 5n\sum_{k=1}^{n} 1/k$$

$T(n) = \text{"10"} + 4n(lg(n))^2 + 2T(n/3)$ *

$n = 3^i$???

$\hookrightarrow O(n(lg(n))^2)$

$$\underbrace{T(3^i) - 2T(3^{i-1})}_{r:2 \quad \alpha:1} = \underbrace{3^i \cdot i^2}_{r:3 \quad \alpha:3}\ ???$$

$(r, n)$

$\hookrightarrow (A + Bn + Cn^2 + \ldots)r^n$

Gen pol -1 mult ^ n

No have same root

$3^i i^2 \equiv b^i \quad p(i) \quad r = b \quad \alpha = 1 + d(p(i))$ → degree

2/ ⊛ Complexity order

$$t(n) = \begin{cases} 3n & (n < 4) \\ \underbrace{3t(n-1) - 2t(n-2)}_{r:2\ \alpha:1} + \underbrace{n}_{r:1\ \alpha:3} + \underbrace{3^n}_{r:3\ \alpha:1} & (o.w.) \end{cases} \quad \Omega(3^n)$$

$t(n) = A + Bn + Cn^2 + D2^n + E3^n \longrightarrow \Theta(3^n)$

3/ ⊛ Complexity Order

$$t(n) = \begin{cases} n^2 & (n < 5) \\ 6t(\sqrt{n}) - 8t(\sqrt[4]{n}) + 16 & (o.w.) \end{cases}$$

$n = a^{2^i} = 2^{(2^i)} \Rightarrow t(2^{(2^i)}) - 6t(2^{(2^{i-1})}) + 8t(2^{(2^{i-2})}) = 16$

$r = 4 \quad \alpha = 1$

$r = 2 \quad \alpha = 1$

$r = 1 \quad \alpha = 1$

$t(2^{(2^i)}) = A + B \cdot 2^i + C4^i \xRightarrow{V.C} t(n) = A + B lg_2(n) + C(lg_2(n))^2 \in O((lg(n))^2)$

## Case 4

$$T_{34}(n) = 10 + 5n + 2 \quad T_{34}(n/2)$$

$$n = 2^i \longrightarrow \underbrace{T_{34}(2^i) - 2T_{34}(2^{i-1})}_{r=2 \quad \alpha=1} = \underbrace{10 + 5 \cdot 2^i}_{\substack{r=1 \quad \alpha=1 \\ r=2 \quad \alpha=1}}$$

$$T_{34}(2^i) = A \cdot 1^i + (B + Ci)\, 2^i$$

$$T_{34}(n) = A + Bn + C \quad lg_2(n)\,n \in O(n\, lg(n)) \qquad \in \Omega(n) \qquad \Theta(n\, lg(n))$$

$\lfloor \text{express} \rfloor$

↳ floor → greatest small number (integer/whole)

$\lceil \text{express} \rceil$

↳ roof

## Case 5

$$T_{35}(n) = \begin{cases} \text{"2"} & n \leq 4 \\ 5 + \sum_{k=1}^{n/2} (7 + O(n)) + 2(5 + T_{35}(n/3)) \end{cases}$$

n = 1000

J = n/k

Last value of k = 500; if k = 50 ½ ...

1000

7·500

$$5 + 7/2\, n + n/2 \cdot On + 10 + 2T_{35}(n/3) = T_{35}(n)$$

$$\hookrightarrow 5 + 7/2\, n + kn^2 + 10 + \underbrace{2T_{35}(n/3) \geq T_{35}(n)}_{n=3^i}$$

$$r=2 \quad \alpha 1$$

$$\underbrace{15}_{r=1\, \alpha=1} + \underbrace{4 \cdot 3^i}_{r=3\, \alpha=1} + \underbrace{k(3^i)^2}_{r=9\, \alpha=1} = T_{35}(3^i) - 2T_{35}(3^i - 1) \; \substack{r=2 \\ \alpha=1}$$

$$T_{35}(3^i) = A1^1 + B \cdot 2^i + C3^i + D9^i$$

$$T_{35}(n) = A + B? + Cn + Dn^2$$

$$2^i = (3^{log(2)})^i = (3^i)^{log(2)}$$

$$\simeq 0'7$$

## Case 6

$$T_{36}(n) \begin{cases} 3 & (n < 1) \\ 5 + T_{36}\left(\frac{n}{3}\right) + n \cdot O(n^2) + lg_3(n)(6 + O(lg(n))) \end{cases}$$

$n = 3^i$

$$\underbrace{T_{36}(3^i) - T_{36}(3^i - 1)}_{r=1 \quad \alpha=1} \leq 5 + \boxed{k_1 \cdot (3^i)^3} + 6i + k_2 i^2$$

↳ r = 27 α = 1

r = 1 α = 3

r = 1 α = 4

r = 27 α = 1

$$T(3^i) = A + Bi + Ci^2 + Di^3 + E\, 27^i$$

$K_1 = 5MB$

$K_2 : 10MB$

$K_3 : 8MB$

$K_4 = 1MB$

$$M(i,j) = \begin{cases} j & (i==0) \\ M(i-1,j) & (K(i+1)\,\mathrm{DIV} > j) \\ \text{Minimum} & \{M(i-1,j), M(i-1, j-K(i+1\,\mathrm{DIV}2))\} \; (o.w) \end{cases}$$

Storage device capacity: 18 MB

M(i,j) = Minimum amount of non-used room in a "j" sized USB full of files from "i"

Rows (Stages) : Ø, }f1{, }f1, f2{, }f1, f2, f3{, }f1, …, f8{
i=0 i=1 i=2 i=8

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ø | 0 | 1 … | | | | 5 NOT SAME | | | | | | | | | | | | | 18 |
| f1 | 0 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 | … | 5 NOT SAME | | | | | | | | 13 |
| f1 f2 | 0 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 | 4 | 0 | 1 … | | | | | | | |
| f2… f3 | 0 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 NOT SAME |
| f1…f4 | 0 | 1 | 2 | 3 | 4 | 0 | … | | | | … | | | | | | 1 | 2 | ⓪ * |
| f1…f5 | | | | | | | | | | | | | | | | | | | * |
| f1…f6 | | | | | | | | | | | | | | | | | | | |
| f1… f7 | | | | | | | | | | | | | | | | | | | |
| f1… f8 | | | | | | | | | | | | | | | | | | | |

F1 F2 F5

Change problem

MD(i,j) : Minimum difference between changes provided and actually one

$$MD(i,j) \begin{cases} j & i == 0 \\ \\ \min \}m(i-1,j), m(i-1, j-\text{value}(i^*))\{ \end{cases}$$

Increasing array, máx num of par so that their product is even. Only One

[3, -1, -1, 0, 2, 17]

Candidates: Index of arrays
Order: increasing
Feasibility: if (A[i] % 2 == 0)} Even++{
Ending condition: (i == size) then return min} Even, size/2{

Solution: 1dimentional tuple B with as many elements as W, sum=0
Exhaustivity B(i) rages in 0-> not set 1-> not included 2-> included 3-> BT
BT Node: B(i) == 0; i--; if (B == 2) } Sum = Sum - W(i); {

Dead Node: (i == last) && [(Sum + [B(i) == 2) ? S(i) : 0]) =! = S
Live node: (Dead node)' if(B(i) == 2)} Sum = Sum + W(i); {
Solution node: (i == last) && (Live node) \ if (B(i) == 2) } Sum = Sum - W(i){: Set B(i) to deadnode

Solution: A 1dimentional array tuple B of size S
Exhaustivity: B(i) in range 0-> not used 1-> not accepted 2-> accepted 3-> BT
Bactracking node: if (B(i) == 0); i--; if (B(i) == 2)} Sum = sum - W(i){;
Dead node: (i == last) && ([Sum + [B(i) == 2] ? S(i) : 0]) =! = S);
Live node: (Dead node)' && (B(i) == 2) } Sum = Sum + W(i));
Solution node: (i == last) && (Live node) \ (B(i) == 2)} Sum = Sum - W(i); Set B(i) to dead node
Solution: One dimension array B of size S
Exhaustivity: B(i) in range 0-> not used 1-> rejected 2-> accepted 3-> BT
BT Node: (B(i) == 0)} i--; if (B(i) == 2)} Sum = Sum - W);
Dead node: (i == last) && ((Sum + [(B(i) == 2)? S(i) : 0)] =! = S
Live node: (Dead node)' \ if B(i) == 2)} Sum = Sum + W(i);
Solution node: (i == last) && Live node \ if (B(i) == 2)} Sum = Sum + W(i): Set B(i) to deadnode

Examen ejemplos

1. Pide Greedy
   - Candidates
   - Order // Increasing, decreasing, none
   - Feasibility // Condición que debe cumplir nuestro candidato para entrar en el proceso
   - Selection
   - Ending condition // ¿Cuándo veo que llegué al último?

⇒ Idea de nodos interconectados $\overset{NI}{}$, array para encontrar X máximo/mínimo

✧ Translators (NI)

↳ Two-way translators, minimize the amount to pay per each pair, even if we have to pass from more than 2 translators

Candidates: Translators

Order: Increasingly amount to spend

Feasibility: Connection made when decreases the amount of unconnected lenguages

Initially all of them are unconnected

Selection: —

Ending condition: When each translator is in the component made in the cheapest way

"When there is only one component / n-1 accepted"

✧ Earthquakes ESII

↳ Cable too expensive. Given distance between computer, all must reach eachother

Candidates: Computers in ESII

Order: Increasing order according to the distance

Feasibility: When the unconnected PCs get connected to other one, decreasing the unconnecteds

Initially all of them are unconnected

Selection —

Ending condition: When all of them are interconnected in the cheapest way