# Lab10 - ANOVA and Kruskal-Wallis Tests

For comparing more than 2 samples we will use the two tests below

- ANOVA: ANalysis Of VAriance: *collection of Statistical Models.* Different ways of making it:
  - One way ANOVA

    First check assumptions:

    - K samples are independent
    - Samples follow **normal distribution**
    - Common variance o^2

    It works by doing:

    - Compute mean for each sample
    - Compare variation of each observation within the sample with sample of mean
    - Compute mean for observations
    - Compute variation of each sample with general mean
    - Compare variance within samples with their variance

    If the sample means are different the the variance within samples must be small compared to the variance between the samples
    $H0$: $\tilde{\mu}1$ = $\tilde{\mu}2$ = ... = $\tilde{\mu}n$
    $H1$: Not every $j\mu$ are equal

    - Equations

      B is num of groups

      Nj is sample size of group j

      N total sample size

      Xij is the ith element of group j

      - Variation between groups
        - Degrees of freedom

          b-1

- Sum of squares

$$SSTbetween = b\sum(j=1) = nj(\bar{X}j - \bar{X})2$$

- Mean of squares

$$MT = SST/b-1$$

▼ Variation within groups

- Degrees of freedom

  n-b

- Sum of squares

$$SSEwhithin = SST - SSTr = b\sum j = 1nj\sum i = 1nj(Xij - \bar{X}j)^2$$

- Mean of squares

$$ME = SSE/n-b$$

▼ Total variation

- Degrees of freedom

  n-1

- Sum of squares

$$SSTtotal = SST = b\sum j = 1nj\sum i = 1nj(Xij - \bar{X}))^2$$

▼ F and Hypothesis testing

This test follows Senedecor's F distribution

Fs=MT/MD

Our test will compare Fs agains Falfa

$$FS > F\alpha; (b-1); (n-b) \rightarrow RejecttorejectH0$$

$$FS <= F\alpha; (b-1); (n-b) \rightarrow FailtorejectH0$$

▼ Example

We will use the mtcars database and we will compare mpg vs gear.

Dice que el va a testear la normality pero dice que si usamos eso no tendremos tiempo en el examen

```
shapiro.test(mtcars$mpg[mtcars$gear == 3])
#Con esto testeamos la normality de todos los samples, cambiando el

bartlett.test(mtcars$mpg ~ factor(mtcars$gear))
#Con esto si tienen la same variance, testeamos la homocedasticity.
#El sample pasa el test ya que no hay diferencia de varianzas

ANOVA <- aov(mtcars$mpg ~ factor(mtcars$gear))
summary(ANOVA)
#Despues de las dos comprobaciones ya podemos hacer anova
#Importante: primero se hace el test y luego el summary
El Pr(>F) es el pvalue.  Podemos ver que es menor  de 0.05 lego rea
nula  de que  no estan asociados el gear y los mpg. Sí lo  estan
```

Dice que habria que hacer una funcion para describir mpg por el mismo y en relacion con el gear number.

```
var n missing min max mean sd median IQR nor.test nor.p.value
mpg 32 0 10.4 33.9 20.09 6.03 19.2 7.375 S-W 0.123
gear var n missing min max mean sd median IQR nor.test nor.p.value
3 mpg 15 0 10.4 21.5 16.11 3.37 15.5 3.900 S-W 0.663
4 mpg 12 0 17.8 33.9 24.53 5.28 22.8 7.075 S-W 0.208
5 mpg 5 0 15.0 30.4 21.38 6.66 19.7 10.200 S-W 0.461
```

▼ Post-hoc.

Viendo los resultados de lo anterior, dice que deberiamos preguntarnos son todos los grupos diferentes delrsto o gear 4 and 5 son similares. Para eso se usa el post-hoc, testea cuales grupos son statiscally different.

Usaremos el Tukey's Honestly Significant Difference.

```
TukeyHSD(ANOVA)
Tukey multiple comparisons of means
    95% family-wise confidence level
Fit: aov(formula = mtcars$mpg ~ factor(mtcars$gear))
$`factor(mtcars$gear)`
            diff       lwr      upr      p adj
```

```
4-3 8.426667 3.9234704 12.929863 0.0002088
5-3 5.273333 -0.7309284 11.277595 0.0937176
5-4 -3.153333 -9.3423846 3.035718 0.4295874

#Comprobamos que no hay diferencias entrre 4-5 y tb entre 3-5 ya que
tb es mayoor de 0.05
```

- Kruskal-Wallis

  Esto se usa cuando las ANOVA assumptions are not met. Es la version no parametrica de One-Way ANOVA

  ```
  kruskal.test(mtcars$mpg ~ factor(mtcars$gear))

  Kruskal-Wallis rank sum test

  data: mtcars$mpg by factor(mtcars$gear)
  Kruskal-Wallis chi-squared = 14.323, df = 2, p-value =
  0.0007758
  #El p value es bellow 0.05 asi que hay diferencias
  #entre el fuel consumption among groups
  ```

  - Post-hoc. Sabemos que son diferentes, pero no sabemos que grupos exactamente. Se usaba el Tukey HDS for ANOVA y para el kruskal usamos el dunns tests, esta en el FSA package

    ```
    FSA::dunnTest(mtcars$mpg ~ factor(mtcars$gear))
    Comparison Z P.unadj P.adj
    1 3 - 4 -3.761746 0.0001687311 0.0005061932
    2 3 - 5 -1.645621 0.0998418909 0.1996837817
    3 4 - 5 1.140586 0.2540422171 0.2540422171
    ```

- CAUTION

  > Dice que cuando usas parametric tests los resultados se pareceran mucho.

Non-parametric tests are a solution for the shortcomings of some
tests if the data does not follow a normal distribution. In that case the parametric test can be over or infra rejecting the null hypothesis and the non-parametric tests “solve” the problem, but if the data is fit for a parametric test the non-parametric test should most of the time show a very similar result.

> Pero no siempre elegimos non parametric aunque siempre funcionen ya que los parametric tests tienen mas statistical power, son mas de fiar y preferibles.

> Solo usaremos non parametric cuando la data desestabilice the parametric tests, O M G SO NASTY