Theme 4. Electric Field

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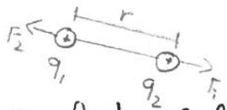
Coulomb (C)
$1nC = 10^{-9}C$
$1\mu C = 10^{-6}C$
$1mC = 10^{-3}C$

1. Coulombs Law



$$\boxed{\vec{F} = k \cdot \frac{|q_1 \cdot q_2|}{r^2} \vec{u}}$$

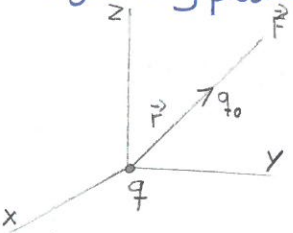
$$\vec{u} = \frac{r_2 - r_1}{|\vec{r}_{21}|}$$

$$k = \frac{1}{4\pi\varepsilon_0}$$

$e = 1'602 \cdot 10^{-19}C$
$k = 9 \cdot 10^9 \, N \cdot m^2/C^2$
$\varepsilon_0 = 8'85 \cdot 10^{-12} \, C^2/N \cdot m^2$

2. Electric field and principle of superposition

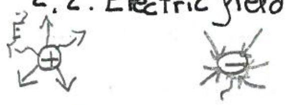
Charged body produces an electric field ; Not necessary to be in contact



$$\boxed{E = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}} \; (N/C)$$

$$\vec{E} = \sum_i k \cdot \frac{q}{r^2}$$

2.2. Electric field lines



Linear
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \lambda \cdot \frac{dl}{r^2}$$

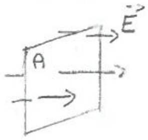
Surface
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \sigma \cdot \frac{ds}{r^2}$$

Volume
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \rho \cdot \frac{dv}{r^2}$$

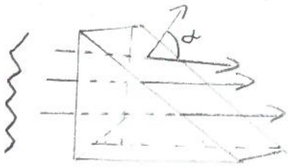
$\lambda = \frac{dq}{dl}$
$\sigma = \frac{dq}{ds}$
$\rho = \frac{dq}{dv}$

3. Electric Flux

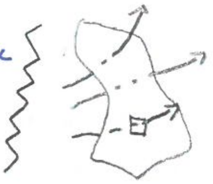
$\Phi = \int \vec{E} \cdot d\vec{S}$ $(N \cdot m^2/C)$



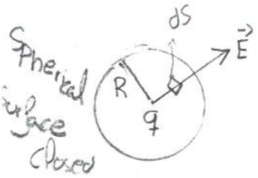
$\Phi = \vec{E} \cdot \vec{S} = E \cdot A$



$\Phi = E \cdot A \cdot \cos\alpha$



$\Phi = \int E \cdot ds$

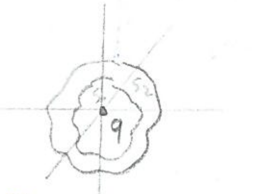


$$\Phi = E \cdot ds = \frac{q}{\cancel{R^2}} \cdot \cancel{4\pi R^2} \cdot \frac{1}{\cancel{4\pi}\varepsilon_0} = \frac{q}{\varepsilon_0}$$

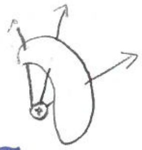
Volume sphere $= \frac{4}{3}\pi r^3$

$$\vec{E} = k \cdot \frac{q}{x_0(x_0 - L)}$$

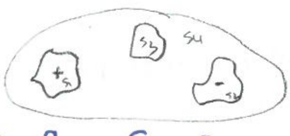
4. Gauss Law



$\Phi_1 = \Phi_2 = \Phi_3 = \ldots$



$\Phi = 0$



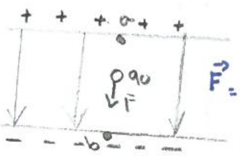
$S_1 = \frac{q}{\varepsilon_0}$ $\quad S_3 = 0$
$S_2 = -\frac{q}{\varepsilon_0}$ $\quad S_4 = \frac{(q-q)}{\varepsilon_0}$

$$E \cdot A_0 = \frac{q_{int}}{\varepsilon_0}$$

Theme 5. Electric potential

1. Electric potential energy

$$\boxed{\begin{array}{l} W_{ab} = -(U_B - U_A) = -\Delta U \\ W_{ab} = -\Delta U = q_0 \cdot E \cdot d \end{array}}$$



$\vec{F} = q_0 \cdot \vec{E}$

$W < 0$; done by the electric field
$W > 0$; done by an external force

2. Electric potential Gradient

$W_{ab} = F \cdot d = q_0 \cdot E \cdot d = -\Delta U_{ab}$

$V = \frac{U}{q}$; $\Delta V = V_B - V_A$ $(J/C = Volt)$

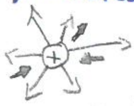
V: potencial eléctrico $\qquad$ E: campo eléctrico

The electric field is not uniform so: $W_{ab} = q_0 \cdot \int_A^B \vec{E} \cdot d\vec{r}$

$E = -\vec{\nabla} V$; $E_p = q_0 \cdot V$

The field lines show the direction in which the potential energy will decrease V_{high}  V_{low}

a) V increases inward $\qquad$ b) V decreases inward



Positive charges $\rightarrow \Delta V < 0$ & $\Delta E_p < 0$
Negative charges $\rightarrow \Delta V > 0$ & $\Delta E_p < 0$

3. Electric potential of a point charge

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{r}$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}}$$

$\rightarrow$ Potential energy
$$U = q_0 \cdot V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \cdot q_0}{r}$$

W from ∞ to the final position

4. Electric potential of a distribution of charges

✓ Discrete distribution

$$V=\sum_n V_n=\frac{1}{4\pi\varepsilon_0}\cdot\sum_n\frac{q_n}{r_n}$$

✓ continuous distribution

$$V=\int dV=\frac{1}{4\pi\varepsilon_0}\cdot\int\frac{dq}{r}$$

6. Equipotential surfaces

$dW=-\vec{F}\cdot d\vec{r}$

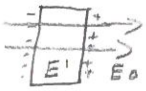
$\rightarrow V_A=V_B\Rightarrow W_{AB}=0$

$\Delta V=-\vec{E}\cdot\Delta\vec{r}$ $\begin{cases}\Delta\vec{r}\text{ perpendicular to }\vec{E}\Rightarrow\Delta V=0\\ \Delta\vec{r}\parallel\text{ to }\vec{E}\rightarrow\text{Maximum variation of potential}\end{cases}$

7. Movement of a particle in an electric field

$F=q\cdot E$; to calculate the acceleration $\Rightarrow\boxed{\Sigma F=m\cdot a}$

8. Conductor in electrostatic equilibrium ($E'=E_0$)



I. Placed on it's surface

↳ $E=0$ inside $\Rightarrow$ No flux $\Phi=0;\ q=0$

II. Electric field at the surface of the conductor is perpendicular to the surface

$\Phi=E\cdot S=\frac{q_{int}}{\varepsilon_0}\Longrightarrow E\cdot\cancel{S}=\frac{\sigma\cdot\cancel{S}}{\varepsilon_0}$

$q_{int}=\sigma\cdot S$ ↳ surface charge density

Theme 6. Capacitors

1. Capacitors and capacitance

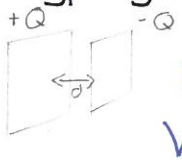
Use: Charge and energy storage in circuits ⇒ electrical capacitance

Construction: Two conductors, with charges $Q+$ and $Q-$, separated by an insulator

$\boxed{C=\frac{Q}{V}}$ (Farads, $F: C/V$)

Dialectric Constant (k)

2. Types of capacitors



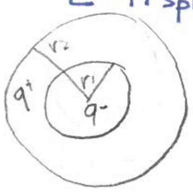
1- Parallel plate

$E=\frac{\sigma}{\varepsilon_0}=\frac{q}{\varepsilon_0 A}$

$V=E\cdot d=\frac{Q}{\varepsilon_0 A}\cdot d$

$C=\frac{q}{V}=\frac{\cancel{q}}{\frac{\cancel{q}}{\varepsilon_0 A}\cdot d}=\boxed{C=\frac{\varepsilon_0 A}{d}}$

2 - A spherical capacitor



$C=\frac{q}{V_{21}}=4\pi\varepsilon_0\cdot\frac{R_1R_2}{R_2-R_1}$

if $R_2\rightarrow\infty\Rightarrow\boxed{C=4\pi\varepsilon_0 R_1}$

3 - Cylindrical capacitor

$C=\frac{q}{V}=\boxed{\frac{2\pi\varepsilon_0 L}{\ln(b/a)}=C}$ ⓐⓑ

3. Networks of Capacitors

3.1 Series

$\boxed{\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}}$; $V=V_1+V_2$

↳ q/V

⊣⊢⊣⊢

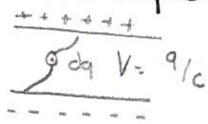
3.2 Parallel

$\boxed{C=C_1+C_2}$

$V=V_{battery}$ $q=q_1+q_2$

4. Energy stored in a capacitor

$V_{(t)}=q^{(t)}/C$



$dW=\frac{q(t)}{C}dq$

$\boxed{W=\frac{q^2}{2C}}$

$\boxed{U=\frac{1}{2}q\cdot V}$

Energy per unit volume

$u_e=\frac{U}{V}=\frac{1}{2}\varepsilon_0E^2$

If we disconnect the capacitor from the battery and introduce the dialectric

$C'=k\cdot C_0$ $\quad E_0=E_0/k$

$q'=q_0$ $\quad U'=U_0/k$

$V'=V_0/k$

$\boxed{C=k\cdot\varepsilon_0\cdot L}$

↳ L for parallel plates: $L=A/d$

↳ L for cylindrical: $L=\frac{2\pi l}{\ln(b/a)}$

If we introduce the dialectric with the battery on:

$C'=k\cdot C_0$ $\quad E_0=E_0$

$V'=V_0$ $\quad U'=k\cdot U'_0$

$q'=q_0\cdot k$

Physics exam 1

Unit 1

Magnitude: phenomena that can be observed and measured
Quantity: State of a magnitude in a given object of phenomenon. → $A/B = n$ ↳ dimentionless

$$B = 1'36 \pm \underbrace{0'04}_{\varepsilon_a}\ T$$

$\varepsilon_a \rightarrow$ $\varepsilon_r(B) = \dfrac{0'04}{1'36} = 0'029 \cdot 100 = 2'9\%$

Base units: Independent quantities and not related by any physical law (time, mass...)
Derived units: through an equation

Plane angle: $\theta = \dfrac{l}{R}$ * l: length of a circular arc
R: radius of the circle

Solid angle: $\Omega = \dfrac{S}{R^2}$

- Scientific notation $X = a \cdot 10^n,\ 1 \leq a \leq 9$
- Order of magnitude n if $a < 5$; $n+1$ if $a \geq 5$
- Significant number

From 1 to 9, all significant numbers

0 → To the left, never
↳ In the middle of two numbers, it is
↳ To the right, may or not may

* Multiplication or division
  ↳ No more significant figures than the starting number with the fewest
* Addition or subtraction

Rounding

Up (5, 6, 7, 8, 9)   Down (1, 2, 3, 4)

1/ People flying rn $\nearrow$ 1/4 ever ; twice a year, $7'7 \cdot 10^9$ people. 4 hours

$$N = \frac{1}{4} \cdot 7'7 \cdot 10^9 \cdot 2 \cdot \frac{4}{365 \cdot 24} = \underbrace{1'8 \cdot 10^6}_{}$$

$OM = 10^6 \Rightarrow 1'8 < 5$; $OM = n$

2/ Whatsapp in Spain in 1 week $\rightarrow$ 1/2 people, 20 times a day

$$N = \frac{1}{2} \cdot 47'1 \cdot 10^6 \cdot 20 \cdot 7 = \underbrace{3'2 \cdot 10^9}_{OM = 10^9}$$

1/ $Log_{10}\left(\frac{3'14}{0'52}\right)$? obtained in lab. Significant figures

$Log_{10}\left(\frac{3'14}{0'52}\right) = 0'78\ldots \Rightarrow \underline{0'78}$, because the fewest significant decimals are 2

5/ Orden de magnitud latidos de una persona en 80 años. Suposiciones

70/minute

$70 \cdot 60 \cdot 24 \cdot 365 \cdot 80 = 8064000 \Rightarrow 8 \cdot 10^6$ latidos $\Rightarrow 10^7$ OM because in $a \cdot 10^n$, $a \geq 5$ so

$n = n+1$

4/ Value directional derivate at $P(1,0,1)$ for $U(x,y,z) = x^2 + 2y + \frac{1}{z}$ along $\vec{n} = (1,1,1)$

$$\frac{\partial U}{\partial x}\vec{i} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = 2x + 2 - \frac{1}{z^2} = 2 + 2 - 1 = 2\vec{i} + 2\vec{j} - \vec{k}$$

$$\vec{u} = \frac{(1,1,1)}{|\vec{u}|} = \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \qquad (2,2,-1) \cdot \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\frac{\partial U_x}{\partial x} = 2x$$

$$\frac{\partial U_y}{\partial y} = 2$$

$$\frac{\partial U_z}{\partial z} = -\frac{1}{z^2}$$

Unit 2

Quality/reliability depends on the observer, measuring apparatus, enviroment, measuring system...

$B = 1'36 \pm \underbrace{0'04}_{E_a}\ T$

Absolute error (ε_a) $= |X_{exact} - X_{measured}|$

Relative error (ε_r) $= \dfrac{\varepsilon_a(x)}{X}$

Direct VS Indirect measurements

Sensivity: Smallest value that can be measured by an instrument. The smaller quantity, the more sensivity instrument

Average: $\bar{x} = \dfrac{\sum x_i}{n}$

Absolute error

1- 3 measurements
2- We calculate the accepted value ($\bar{x}$)
3- The range (R) $= |x_{max} - x_{min}|$
4- Calculate the dispersion; $T = 100 \cdot \dfrac{R}{\bar{x}}$

$0 < T \leq 2\% \Rightarrow 3$

$2 < T \leq 8\% \Rightarrow 6$ ☆

$8 < T \leq 12\% \Rightarrow 15$

$T > 12\% \Rightarrow 50$

☆ $\underbrace{\varepsilon_i = |x_i - \bar{x}| \Longrightarrow \varepsilon_{mean}}_{\varepsilon_a(x) = \{\text{sensivity},\ \varepsilon_D,\ \varepsilon_{mean}\}} \Longrightarrow \varepsilon_D = \dfrac{R}{4}$

$X = \bar{X} \pm \varepsilon_a(x)$

$\varepsilon_r = \dfrac{\varepsilon_a(x)}{\bar{X}}$

$A = f(B, C, D) \Rightarrow B \pm \varepsilon_a(B);\ C \pm \varepsilon_a(C);\ D \pm \varepsilon_a(D)$

$$\varepsilon_a(A) = \sqrt{\left(\frac{\partial f}{\partial B} \cdot \varepsilon_a(B)\right)^2 + \left(\frac{\partial f}{\partial C} \cdot \varepsilon_a(C)\right)^2}$$

Only with 1 significant figure (two if the first is a 1 or 2)

2) sensivity = 0'04 mA $\qquad$ $\mathcal{E}_a(x) = max(sensiv, \mathcal{E}_m, \mathcal{E}_D)$

$I_1 = 1'52\ mA \quad I_2 = 1'55\ mA \quad I_3 = 1'54\ mA \quad \mathcal{E}_a\ \&\ \mathcal{E}_r?$

$\mathcal{E}_a(x) = max\{\underline{0'04}, 0'0111, 0'0075\}$ $\qquad$ $I = 1'54 \pm 0'04\ mA$

$\mathcal{E}_D = \frac{R}{4}$ $\qquad$ $\mathcal{E}_r = \frac{\mathcal{E}_a}{\bar{x}} = \frac{0'04}{1'54} = 0'026 = 2'6\%$

$\bar{x} = \frac{1'52 + 1'55 + 1'54}{3} = 1'53\hat{6}\ mA$

$R = |x_{max} - x_{min}| = 1'55 - 1'52 = 0'03$

$T = 100 \cdot \frac{0'03}{1'536} = 1'95\% \leq 2\%$

$\mathcal{E}_1 = ||1'52 - 1'537| = 0'0167\ mA \quad \mathcal{E}_2 = |1'55 - 1'536| = 0'0133\ mA$

$\mathcal{E}_3 = |1'54 - 1'537| = 0'0033\ mA$

$\mathcal{E}_{mean} = \frac{0'0167 + 0'0133 + 0'0033}{3} = 0'0111$

$\mathcal{E}_d = \frac{R}{4} = \frac{0'03}{4} = 0'0075\ mA$

3) sensivity: 0'3°C $\quad T_1 = 25'8°C \quad T_2 = 26'2°C \quad T_3 = 25'6°C \quad \mathcal{E}_a\ \&\ \mathcal{E}_r?$

$T = 100 \cdot \frac{R}{\bar{x}}$ $\qquad$ $R = |x_{max} - x_{min}| = 26'2 - 25'6 = 0'8$

$\bar{x} = \frac{26'2 + 25'8 + 25'6}{3} = 25'8\hat{6} \simeq 25'87$ $\qquad$ $T = 100 \cdot \frac{0'8}{25'87} = 3'09\% \Rightarrow 6$ measurements

$T_1 = |26'2 - 25'87| = 0'33$

$T_2 = |25'8 - 25'87| = 0'07$

$T_3 = |25'6 - 25'87| = 0'27$

$T_4 = |26 - 25'87| = 0'13$

$T_5 = |25'7 - 25'87| = 0'17$

$T_6\ 26'1 - 25'87| = 0'23$

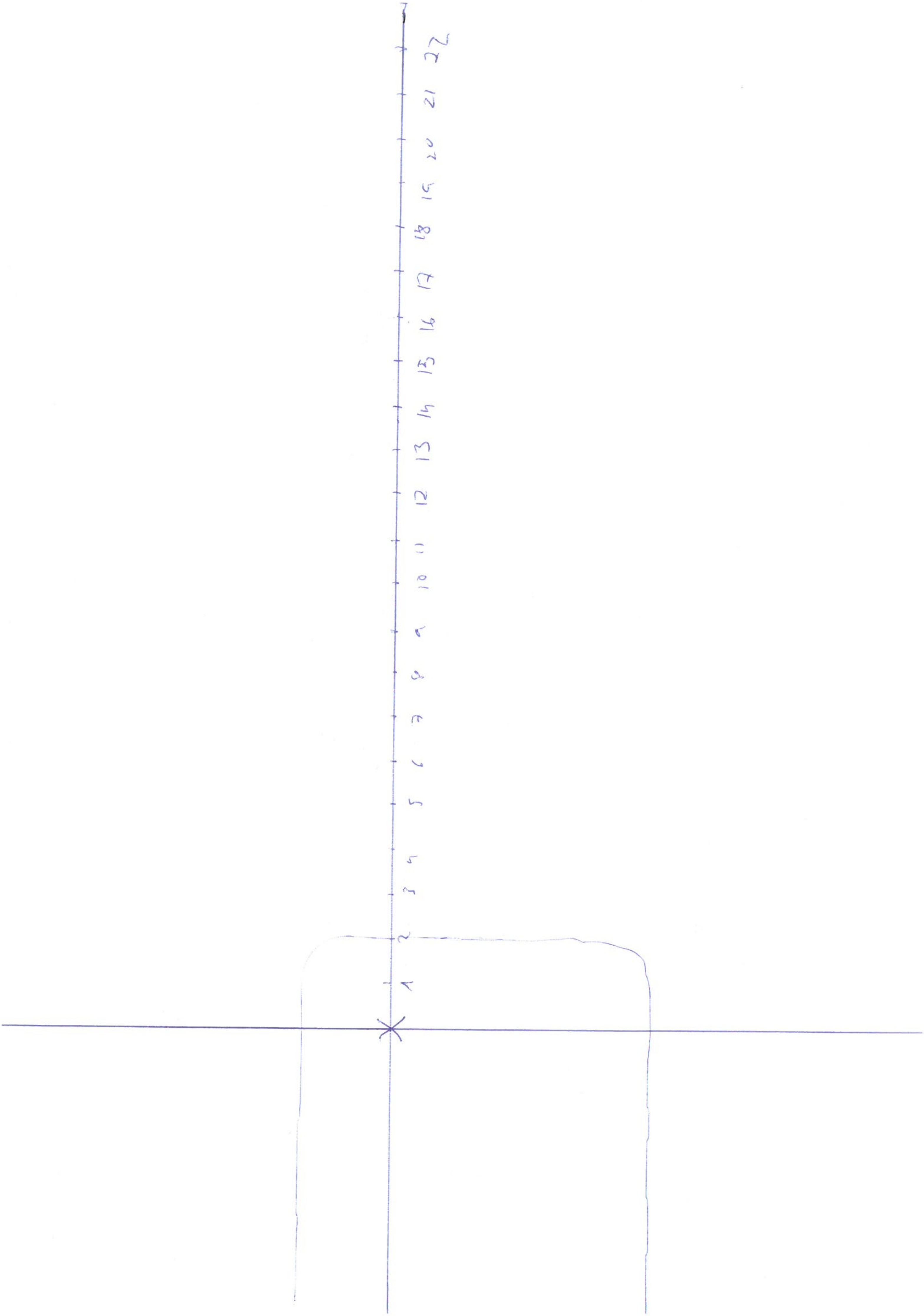
$\mathcal{E}_{mean} = \frac{0'33 + 0'07 + 0'27 + 0'13 + 0'17 + 0'23}{6} = 0'2$

$\mathcal{E}_D = R/4 = 0'8/4 = 0'2$

$T = 25'9 \pm 0'3\ °C$ $\qquad$ $\mathcal{E}_r = \frac{\mathcal{E}_a}{\bar{x}} = \frac{0'3}{25'90} = 0'0115 \cdot 100 = 1'15\%$

$\mathcal{E}_a = max\{$sensivity$, \mathcal{E}_D, \mathcal{E}_{mean}\}$

$\mathcal{E}_a = max\{\underline{0'3}, 0'2, 0'2\}$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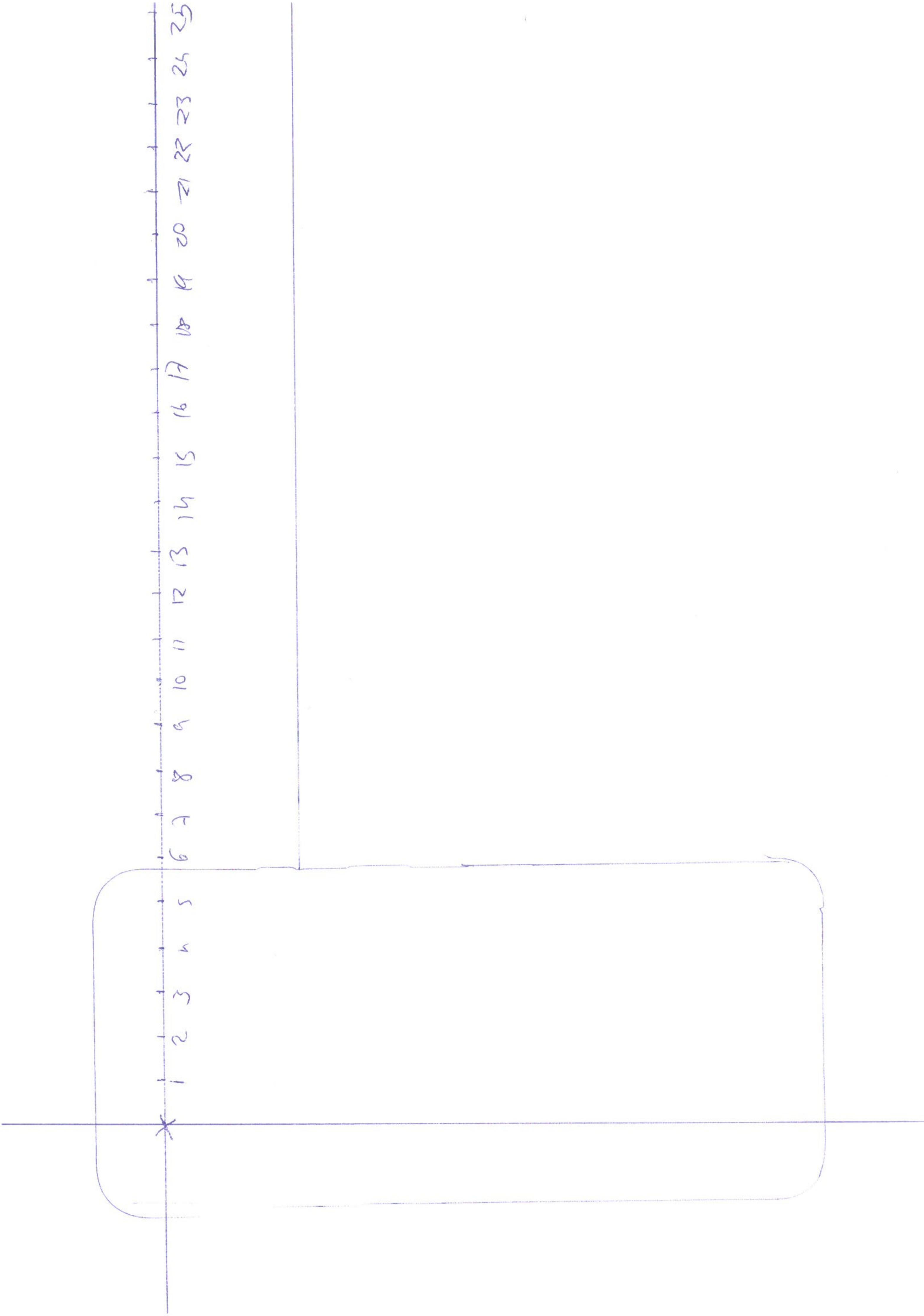
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1/ Divergence of $A(x,y,z) = x^2\vec{i} + (5z - 2xy)\vec{j} + (x^2+y^2)\vec{k}$. Solenoidal?

$$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = 2x - 2x + 0 = \text{Solenoidal} = 0$$

$$\frac{\partial A_x}{\partial x} = 2x$$

$$\frac{\partial A_y}{\partial y} = -2x$$

$$\frac{\partial A_z}{\partial z} = 0$$

2/ $U(x,y) = \frac{1}{2}(x^2 - y^2)$ gradient; ¿harmonic?

$$\vec{\nabla} U(x,y) = \frac{\partial U}{\partial x}\vec{i} + \frac{\partial U}{\partial y} = \frac{1}{2}\cdot(2x) \div \frac{\cancel{1}}{2}\cdot(2y);\ \vec{\nabla} U = x\vec{i} - y\vec{j}$$

Divergence

$$\vec{\nabla}\cdot\vec{F} = \frac{\partial F_x}{\partial F_x} + \frac{\partial F_y}{\partial y} = 1 - 1 = 0 \text{ solenoidal}$$

Curl

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & -y & 0 \end{vmatrix} = \begin{vmatrix} \partial/\partial y & \partial/\partial z \\ -y & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x & -y \end{vmatrix}$$

$= 0 =$ Irrotational

3/ $T(x,y) = \frac{4x}{x+y}$

a) Directional derivated $P(1,1,0)$ along $\vec{n} = (6,-8,0)$

$$\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z} = \frac{4x + \cancel{4x} - \cancel{4x}}{(x+y)^2} = \frac{4y}{(x+y)^2}$$

$$\frac{\partial T}{\partial y} = \frac{-4x}{(x+y)^2}$$

$$\frac{4y}{(x+y)^2}\vec{i} - \frac{4x}{(x+y)^2} \text{ with } P(1,1,0) = \vec{i} - \vec{j}$$

$$u = \frac{(6,-8,0)}{|\vec{u}|} = (0'6, -0'8, 0)\cdot(1,-1) = 0'6 + 0'8 = 1'4$$

b) maximum directional derivate (Gradient)

$$\vec{\nabla} U(x,y) = \frac{\partial U}{\partial x}\vec{i} + \frac{\partial U}{\partial y}$$

$$\frac{4(x+y) - 4x}{(x+y)^2} + \frac{0 - 4x}{(x+y)^2}$$

9) $q = 4'46\ \mu C$ at $(0,0)$ $\quad E = 137\,\hat{i}\ N/C$

through Gaussian sphere radius $R = 20'2$ cm centered at 0,0

$$\Phi = E \cdot dS = \frac{1}{\cancel{4\pi}\varepsilon_0} \cdot \frac{q}{\cancel{r^2}} \cdot \cancel{4\pi r^2} = \frac{q}{\varepsilon_0} = \frac{4'46 \cdot 10^{-6}}{8'85 \cdot 10^{-12}} = 5'03 \cdot 10^{5}\ N \cdot m^2/C$$

10) $r_1 = 9$ cm

$r_2 = 15$ cm $\qquad$ E at 12cm from the center?

$Q = 36\ \mu C$

$$\cancel{E = k \cdot \frac{q}{r^2} = 9 \cdot 10^{9} \cdot \frac{36 \cdot 10^{-6}}{0'12^2}}$$

1) Q and R

$r_1 = 40$ cm

$E_1 =$

5/ $\mathcal{E}_a$ and $\mathcal{E}_r$ V_B $R = 3'79 \pm 0'18 m$ $h = 1'25 \pm 0'06 m$

$V = A_{base} = \pi r^2 \cdot L = \pi \cdot 3'79^2 \cdot 1'25 = 56'4 \, m^3$

$V = 56'40 \pm 3'19 \, m^3$

$\mathcal{E}_r(V) = \sqrt{\mathcal{E}_a(R)^2 + \mathcal{E}_a(h)^2} = \sqrt{0'04^2 + 0'04^2} = 0'056\ldots$

$\mathcal{E}_a(R) = \dfrac{0'18}{3'79} = 0'04$

$\mathcal{E}_a(V) = \mathcal{E}_r(V) \cdot V = 0'056 \cdot 56'4 = 3'19 \, m^3$

$\mathcal{E}_a(h) = \dfrac{0'06}{1'25} = 0'04$

4/ $[x, y]$ $x = 3'6 \pm 0'4 m$, $y = 1'9 \pm 0'3 m$ $\mathcal{E}_a$ $z = x^2 y$

$z = 3'6^2 \cdot 1'9 = 24'6 \simeq 25$

$z = 25 \pm 7 m$

$\mathcal{E}_a(z) \sqrt{\left[\dfrac{\partial z}{\partial x} \cdot \mathcal{E}_a(x)\right]^2 + \left[\dfrac{\partial z}{\partial y} \cdot \mathcal{E}_a(y)\right]^2} = \sqrt{5'472^2 + 3'882^2} = 6'71 \simeq 7$

$\dfrac{\partial z}{\partial x} = 2x \cdot y \cdot \mathcal{E}_a(x) =$

$= 2 \cdot 3'6 \cdot 1'9 \cdot 0'4 = 5'472$

$\dfrac{\partial z}{\partial y} \cdot \mathcal{E}_a(y) = x^2$

$= 3'6^2 \cdot 0'3 = 3'882$

5) Sides ▭ $a = 12'3 \pm 0'4\,m$ $b = 18'74 \pm 0'12\,m$ Area with $\mathcal{E}_a$; $\mathcal{E}_r$

a) With partial derivates (RMS)

$A = a \cdot b = 12'3 \cdot 18'74 = 230'5\,m^2 \simeq 231\,m^2$

$$\frac{\partial A}{\partial a} \cdot \mathcal{E}_a(a) = b \cdot \mathcal{E}_a(a) = 18'74 \cdot 0'4 = 7'496\,m^2$$

$$\frac{\partial A}{\partial b} \cdot \mathcal{E}_a(b) = a \cdot \mathcal{E}_a(b) = 12'3 \cdot 0'12 = 1'476\,m^2$$

$$\mathcal{E}_a(A) = \sqrt{\left[\frac{\partial A}{\partial a} \mathcal{E}_a(a)\right]^2 + \left[\frac{\partial A}{\partial b} \cdot \mathcal{E}_a(b)\right]^2} = \sqrt{7'496^2 + 1'476^2} = 7'640 \simeq 8\,m^2$$

$A = 231 \pm 8\,m^2$ $\mathcal{E}_r(a) = \dfrac{\mathcal{E}_a(x)}{(\bar{x})} = \dfrac{8}{231} = 0'03$

b) With monomial ($\mathcal{E}_r$)

$$\mathcal{E}_r(A) = \sqrt{\mathcal{E}_a(a)^2 + \mathcal{E}_a(b)^2} = 0'0331\ldots \simeq = 0'03$$

$\mathcal{E}_a(a) = \dfrac{0'4}{12'3} = 0'0325$

$\mathcal{E}_a(A) = \mathcal{E}_r(A) \cdot A = 0'03314 \cdot 231 = 7'655 \simeq 8\,m^2$

$\mathcal{E}_a(b) = \dfrac{0'12}{18'74} = 0'0064$

1/ Exam $F = q \cdot v \cdot B$

$q = 0'44 \pm 0'07\,\mu C$ ($\cdot 10^{-6}$, $\cdot 10^{-6}\,C$) $v = 1'25 \pm 0'03\,Mm/s$ ($\cdot 10^6$, $\cdot 10^6$)

$F = 0'44 \cdot 10^{-6} \cdot 1'25 \cdot 10^6 \cdot 3'47 = 1'908\,N$ $B = 3'47 \pm 0'06\,T$

$$\mathcal{E}_r(A) = \sqrt{\mathcal{E}_a(a)^2 + \mathcal{E}_a(b)^2 + \ldots} = \sqrt{0'159^2 + 0'0172^2 + 0'024^2} = 0'1617 \cdot 100 = 16\%$$

$\mathcal{E}_a(q) = \dfrac{0'07}{0'44} = 0'159$

$\mathcal{E}_a(F) = \mathcal{E}_r(F) \cdot F = 0'1677 \cdot 1'908 = 0'308 \simeq 0'3\,m$

$\mathcal{E}_a(B) = \dfrac{0'06}{3'47} = 0'0172$

$\boxed{F = 1'9 \pm 0'3\,N}$

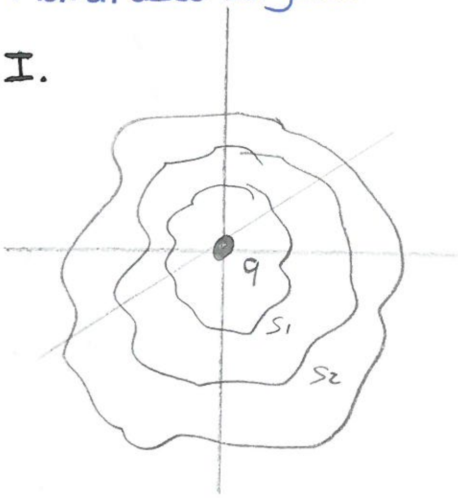
$\mathcal{E}_a(v) = \dfrac{0'03}{1'25} = 0'024$

2/ $\log\left(\dfrac{3'14}{0'52}\right) = 0'78$

4. Gauss's Law

Flux on closed surfaces $\Phi = \frac{q}{\varepsilon_0}$

I.



$\Phi_1 = \Phi_2 = \Phi_3$

II.

$\Phi = 0$

1) $q = 72\,nC$ (0'0) ¿L? $\xrightarrow{x}$ P(12,0) E= 6'0 N/C Linear

$E = \underbrace{\frac{1}{4\pi\cdot\varepsilon_0}}_{k} \cdot \frac{dl}{r^2}$; $l = \frac{E\cdot r^2}{k} = \frac{6\cdot 12^2}{9\cdot 10^9} = 9'6\cdot 10^{-8}$

2) Sphere $r = 8cm = 0'08m$ ¿E at $r = 2'4\,cm$? $0'024m$

charge density $\rho = 489\,nC/m^3$

$489\cdot 10^{-9}\,C/m^3$

$E = k\cdot\rho\cdot\frac{dv}{r^2}$; $E = \frac{1}{4\pi\cdot\varepsilon_0}\cdot\rho\cdot\frac{4\pi r^2}{r^2}$

2) 2 point charge 2 cm ; F=18'1N. to 6'3 cm ¿F?

$F = k\cdot\frac{q\cdot q}{r^2}$; $q\cdot q = \frac{F\cdot r^2}{k} = \frac{18'1\cdot 0'02^2}{9\cdot 10^9} = 8'044\cdot 10^{-13}$

$F = k\cdot\frac{q\,q}{r^2} = 9\cdot 10^9\cdot\frac{8'044\cdot 10^{-13}}{0'063^2} = 1'82$

Theme 4. Electric field

Calombs (C)

$1nC = 10^{-9}C$

$1\mu C = 10^{-6}C$

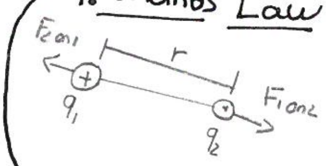
$1mC = 10^{-3}C$

$e = 1'602 \cdot 10^{-19}C$

$K = 9 \cdot 10^{9} N\,m^2/C^2 = \boxed{\dfrac{1}{4\pi \varepsilon_0}}$

$\varepsilon_0 = 8'85 \cdot 10^{-12}\ C^2/N \cdot m^2$

1. Calombs Law



$$\boxed{F = k \cdot \frac{|q_1 \cdot q_2|}{r^2}}$$

$\vec{F_1} = -\vec{F_2}$

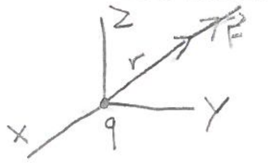
$F_{1on2} = F_{2on1}$

Vectorial expression

$$\vec{F}_{12} = k \cdot \frac{q_1 \cdot q_2}{{r_{12}}^2} \cdot \vec{u}_{12}$$

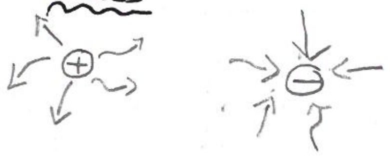
$$\boxed{\vec{u} = \frac{r_2 - r_1}{|\vec{r_{12}}|}}$$

2. Electric field



$$\boxed{E = \frac{F}{q_1} = k \cdot \frac{q}{r^2} \cdot \vec{u}}$$

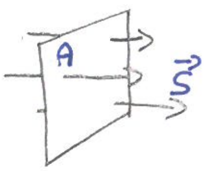
Lines



$$\boxed{\vec{E} = \sum_i k \cdot \frac{q_1}{r^2_{pi}} \cdot \vec{u}_{ri}}$$

3. Electric flux

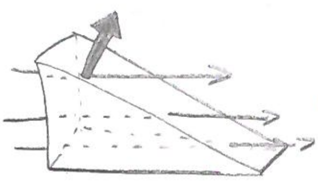
$$\boxed{\Phi = \int \vec{E} \cdot d\vec{S}}$$ $N \cdot m^2/C$



$\Phi = \int E \cdot dS = E \cdot S$

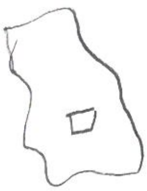
Uniform field

OPEN



$\Phi = \int E \cdot S = E \cdot A \cdot \cos\theta$

Uniform field with angle



$\Phi = \int E \cdot ds$

Non uniform field

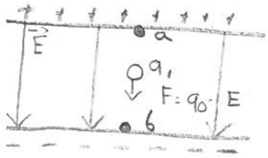
$\Phi = \dfrac{q}{\varepsilon_0}$

CLOSED

3) $\varepsilon_1 = 2'00\varepsilon_0$ $\varepsilon_2 = 3'00\varepsilon_0$ $\varepsilon_3 = 6'00\varepsilon_0$ $d = 6'00\,mm$ $A = 444m^2$

Theme 5. Electric potential

$\cancel{W_{ab} = -\Delta U = -\Delta E_p}$ $W_{ab} = -(U_B - U_A) = -\Delta U$

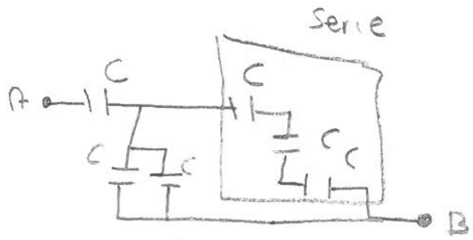


$W_{ab} = -\Delta U = q_0 \cdot E$

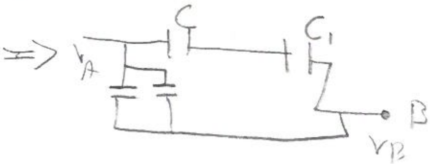
$W_{ab} = F \cdot d = q_0 \cdot E \cdot d = -\Delta U_{ab}$

Electric potential

$V = \frac{U}{q}$ (J/C = Volt) $\Delta V = V_B - V_A = \frac{U_f}{q_0} - \frac{U_i}{q_0} = \frac{\Delta U}{q} = -\frac{W_{ab}}{q_0}$ $V_B - V_A = -E \cdot d$



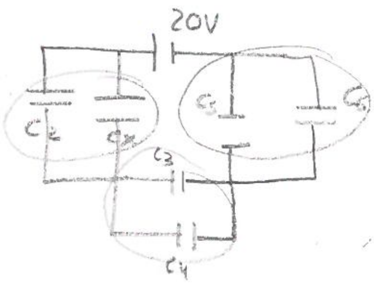
$C = 4 \mu C$



2/ Work to mae $q_0 = -8'0 mC$ to A to B

W

3/ $C_T = 12 \mu F$

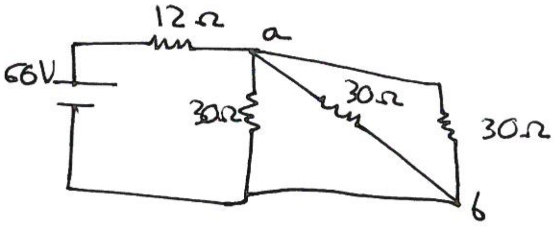


$C_{3-4} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{C_{3-4}}$

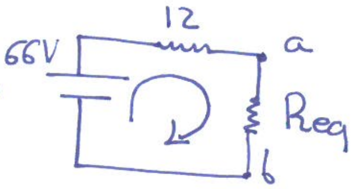
$\frac{1}{C_{3-4}} = \frac{1}{24} + \frac{1}{16} = 1/40$

$C_{56} = \frac{1}{8} + \frac{1}{12} = 1/20$

Potential difference



$\Rightarrow$



$$\frac{1}{R_{eq}} = \frac{1}{30} + \frac{1}{30} + \frac{1}{30} = 10\Omega$$

$$I = \frac{V}{R} = \frac{66}{22} = 3A$$

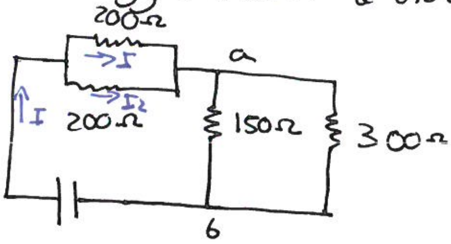
$$V_a - 10I = V_b$$

$$V_a - V_b = 10 \cdot 3 = 30V$$

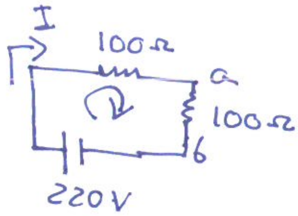
a) Current in each resistor

b) Power in each resistor

c) Pot diff between a and b



$\Rightarrow$



$$-100I - 100I + 220 = 0$$

$$\frac{220}{200} = I_T = 1'1A$$

$$I = I_1 + I_2$$

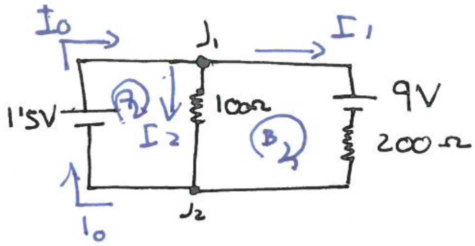
$$1'1 = 2I_1 = 0'5 = I_1$$

$$I = I_3 + I_4$$

$$I = I_3 + 2I_3$$

$$I = 3I_3;\ I_3 = 1'1/3 = 0'3\overline{6};\ I_4 = 0'73A$$

c) $V_a - 150 I_5 = V_b;\ V_a - V_b = 150 \cdot 0'73 = 109'5$



JR: $I_0 = I_1 + I_2$

LR: $-100 I_2 + 1'5 = 0$

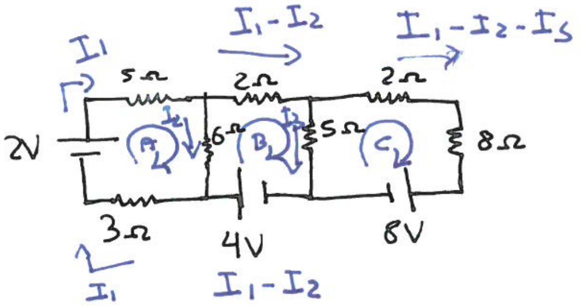
$-9 - 200 I_1 + 100 I_2 = 0$

$$\left.\begin{aligned} I_0 - I_1 - I_2 &= 0 \\ -100 I_2 &= -1'5 \\ -200 I_1 + 100 I_2 &= 9 \end{aligned}\right\} \begin{aligned} I_2 &= 0'015\,A \\ I_1 &= -0'0375\,A \\ I_0 &= -0'0225\,A \end{aligned}$$

$V_{100} = 0'015 \cdot 100 = 1'5V$

$V_{200} = 7'5V$

$P_{100} = I \cdot V = 0'015 \cdot 1'5 = 0'0225W$



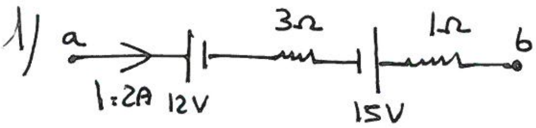
A: $-5I_1 - 6I_1 - 3I_1 + 2 = 0$

B: $-2(I_1 - I_2) - 5I_3 + 4 + 6I_2 = 0$

C: $\underbrace{-2(I_1 - I_2 - I_3)}$ $- 8(I_1 - I_2 - I_3) - 8 + 5I_3 = 0$

$$\left.\begin{aligned} 8I_1 + 6I_2 &= 2 \\ 2I_1 - 8I_2 + 5I_3 &= 4 \\ -10I_1 + 10I_2 + 15I_3 &= 8 \end{aligned}\right\} \begin{aligned} I_1 &= 0'25\,A \\ I_2 &= 0A \\ I_3 &= 0'7\,A \end{aligned}$$

$R(\Omega)$	$I(A)$	$V(V)$	$P(W)$
5	0'25	–	–
2	0'25	–	–
2	-0'45	–	–
3	0'25	–	–
6	0		–
5	0'7	–	–
8	0'45		

1)  Potential? (Voltage)

$V_a - 12 - 3I + 15 - 1I = V_b$; $V_a - V_b = +12 + 3 \cdot 2 + 15 + 2 = 5V$

2) Current by ammeter, direction? 

$I = \frac{V_a - V_b}{R_{eq}} = \frac{28 - 0}{7} = 4A$ $\quad \frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{6} : 2\Omega + 5 = 7\Omega$

b) Induced emf?

$$\mathcal{E} = -\frac{\Delta \Phi}{\Delta t} = -\frac{0'65}{0'023} = -28'06$$

c) Induced current R = 20 ohms

$$I_{ind} = \frac{|\mathcal{E}|}{R} = \frac{28'06}{20} = 1'403 \text{ A}$$

2/ 20 loops from +2 Wb to -3 Wb in 425 ms

a) I emf?

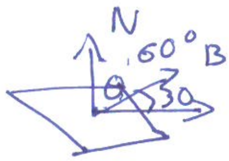
$$\mathcal{E} = -\frac{-3-2 \cdot 20}{0'425} = 235'29$$

b) Resistance if I = 5'12 A?

$$I = \frac{\mathcal{E}}{R}; \quad R = \frac{\mathcal{E}}{I} = \frac{235'29}{5'12} = 45'95 \, \Omega$$

3/ 150 loops. From 5cm × 8cm to 7cm × 11cm in 0'15 s in B = 25 T for 30°

a) $$\mathcal{E}_{ind} = -\frac{N \Delta \Phi_B}{\Delta t} = \frac{-N \cdot B \cdot \Delta A \cdot \cos\theta}{\Delta t} =$$



$$= \frac{-150 \cdot 25 \cdot (0'07 \cdot 0'11 - 0'05 \cdot 0'08) \cdot \cos 60}{0'15} = -46'25 \text{ V}$$

4/ 15cm × 20cm 25 loops angle between normal line and the B (B) changes from 70 to 30° in 85 ms

a) $\mathcal{E}_{ind}$?

$$\mathcal{E}_{ind} = -\frac{N \cdot \Phi \, \Delta\cos\theta}{\Delta t} = \frac{-25 \cdot 3 \cdot \overset{0'15 \cdot 0'2}{\cdot} (\cos 30 - \cos 70)}{0'085} = -13'87 \text{ V}$$

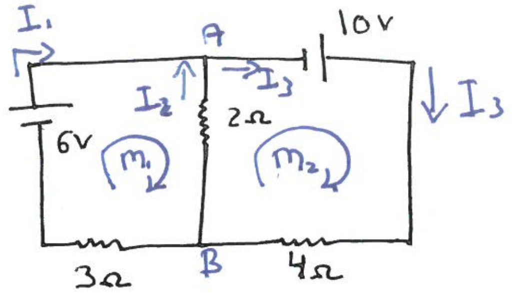
5/ l = 0'45 m v = 2 m/s B = 8 T

a) $\mathcal{E}$?

$$|\mathcal{E}| = B \cdot l \cdot v = 8 \cdot 0'45 \cdot 2 = 7'2 \text{ V}$$

b) B in the rod?

1)

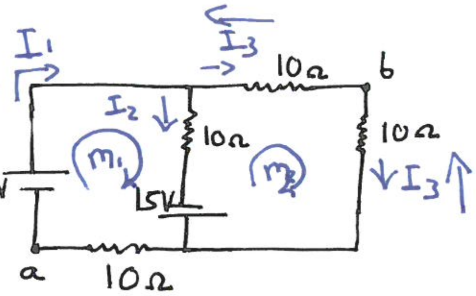


$$I_1 + I_2 - I_3 = 0$$
$$6 = -2I_2 + I_3$$
$$10 = 2I_2 + 4I_3$$

$$\begin{aligned} I_1 + I_2 - I_3 &= 0 \\ -2I_2 + I_3 &= 6 \\ 2I_2 + 4I_3 &= 10 \end{aligned}$$

$I_1 = 2'15A$
$I_2 = 0'23A$
$I_3 = 2'38A$

2)



$$I_2 + I_3 - I_1 = 0$$
$$10 + 15 = 10I_2 + 10I_1$$
$$-15 = 10I_3 + 10I_3 - 10I_2$$

$$\begin{aligned} -I_1 + I_2 + I_3 &= 0 \\ 10I_1 + 10I_2 + 0 &= 25 \\ -10I_2 + 20I_3 &= -15 \end{aligned}$$

$I_1 = 1'2$
$I_2 = 1'3$
$I_3 = -0'1 \Rightarrow 0'1$

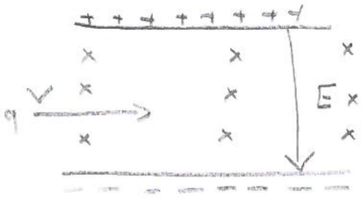
$R_T = 10 + 10 = 20$

$$\frac{1}{R_T} = \frac{1}{20} + \frac{1}{10} = 6'\hat{6} + 0 = 16'\hat{6}$$

$V_a = I_1 \cdot R_T = 16'\hat{6} \cdot 1'2 = 20$

$V_b = 0'1 \cdot 16'6$

1) $B = 1'25\,T$ $E = 195\ kV/m = 195.000\ N/C$



Velocity selector

$q \cdot v \cdot B = q \cdot E\ ;\ v = E/B = \frac{195.000}{1'25} = 156.000\ m/s$

2) $\vec{B} = 1'37\,\vec{i}\ T$

$\vec{V} = 3140\,\vec{i} + 4200\,\vec{j}\ km/s$ Sign and value of q?

$\vec{F} = 7'42\,\vec{k}\ N.$

$\vec{F} = q \cdot \vec{V} \times \vec{B}$

$$\vec{V} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3140 & 4200 & 0 \\ 1'37 & 0 & 0 \end{vmatrix} = (0,\ 0,\ -5'75 \cdot 10^6)\,\vec{k}$$

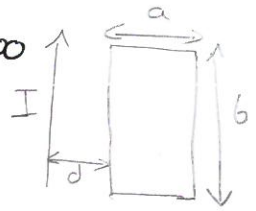
$q = 7'42 / -5'75 \cdot 10^6 = -1'28 \cdot 10^{-6}\ C$

4) Rectangular conducting coil. $N = 8000$

$a = 24\,cm$

$b = 36'00\,cm$ $I = 5'5A$

$d = 6\,cm$



$dS = b \cdot dx$

a) Magnetic flux?

$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{x}$:

$$\phi = N \cdot \int B \cdot dS = N \cdot \frac{\mu_0 \cdot I \cdot b}{2\pi} \int_d^{d+a} \frac{1}{x}\,dx = N \cdot \frac{\mu_0 \cdot I \cdot b}{2\pi} \cdot \ln x \Big|_d^{a+d} = 5'1 \cdot 10^{-3}\,Wb$$

b) ~~During 2'44s current from 8'5 to 1'10A emf?~~ No idea Si idea

$$\mathcal{E} = -\frac{\Delta\phi}{\Delta t} = -\frac{\phi_2 - \phi_1}{2'44} = -\frac{1'02 \cdot 10^{-3} - 5'1 \cdot 10^{-3}}{2'44} = 1'7 \cdot 10^{-3}\,V$$

5) $R = 10'14\,\Omega$ $\phi_m(t) = 8 + 22t^2 - 12t^3$

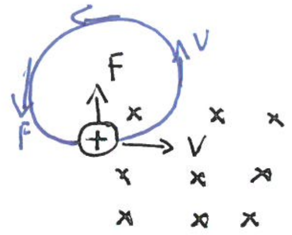
max induced

$\mathcal{E}_{ind} = -\frac{d\phi}{dt} = -(44t - 36t^2) = -44t + 36t^2$

$I_{ind} = \frac{\mathcal{E}}{R} = \frac{-44t + 36t^2}{10'14}$

$dI_{nd}/dt = 0$

$\frac{(-44 + 72t)}{10'14} = 0\ ;\ t = 0'61\,s$

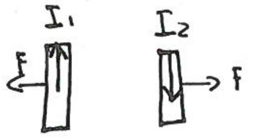


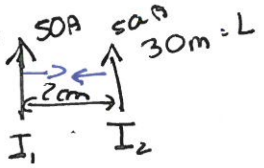
$$R = \frac{m \cdot v}{q \cdot B}$$

1) $v = 5 \cdot 10^6 \, m/s \perp$ $B = 2'5 \, T$ ⊕

a) R?

$$R = \frac{m \cdot v}{q \cdot B} = \frac{1'637 \cdot 10^{-27} \cdot 5 \cdot 10^{6}}{1'6 \cdot 10^{-19} \cdot 2'5} = 0'0209 m$$



1) 

$$F = \frac{I_1 I_2 L \mu_0}{2\pi \cdot r} = \frac{50 \cdot 50 \cdot 30 \cdot 4\pi \cdot 10^{-7}}{0'02 \cdot 2\pi} = 0'75 \, N$$

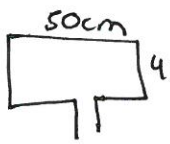
2) Solenoid 15cm and 800 turns wire I=5 B at center?

$n = N/l = 800/0'15 = 5'333$ turn/m

$B = n \cdot \mu_0 \cdot I = 5'333 \cdot 4\pi \cdot 10^{-7} \cdot 5 = 0'0335 T$

3)  r: 30cm, 50 loops   A = 8A   B = 5T   max torque?

$T = 50 \cdot 8 \cdot (\pi \cdot 0'3)^2 \cdot 5 = 565'5 T$

4) 

200 loops I = 15A B to max torque of 1200 N·m

$T = NIAB$; $B = \frac{T}{NIA}$; $B = \frac{1200}{200 \cdot 15 \cdot 0'4 \cdot 0'5} = 2T$

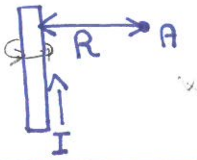
1) 1 loop R = 2'5cm   1'5T to 4'8T in 23ms = 0'023s

a) change in mag flux

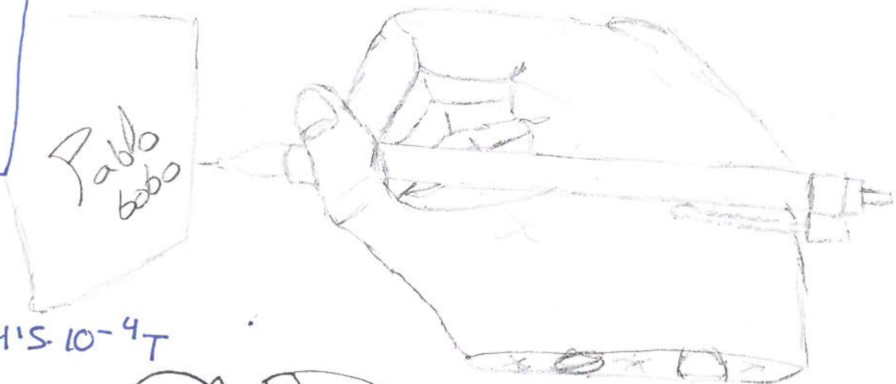
$$\Phi_T = \frac{\Phi_2 - \Phi_1}{\Delta t} = \frac{0'94 - 0'29}{0'023} = \frac{0'65}{0'23}$$

$\Phi_2 = NBA = 1 \cdot 4'8 \cdot \pi \cdot 0'25^2 = 0'94$

$\Phi_1 = NBA = 1 \cdot 1'5 \cdot \pi \cdot 0'25^2 = 0'29$



$$B = \frac{\mu_0 \cdot I}{2\pi \cdot R}$$

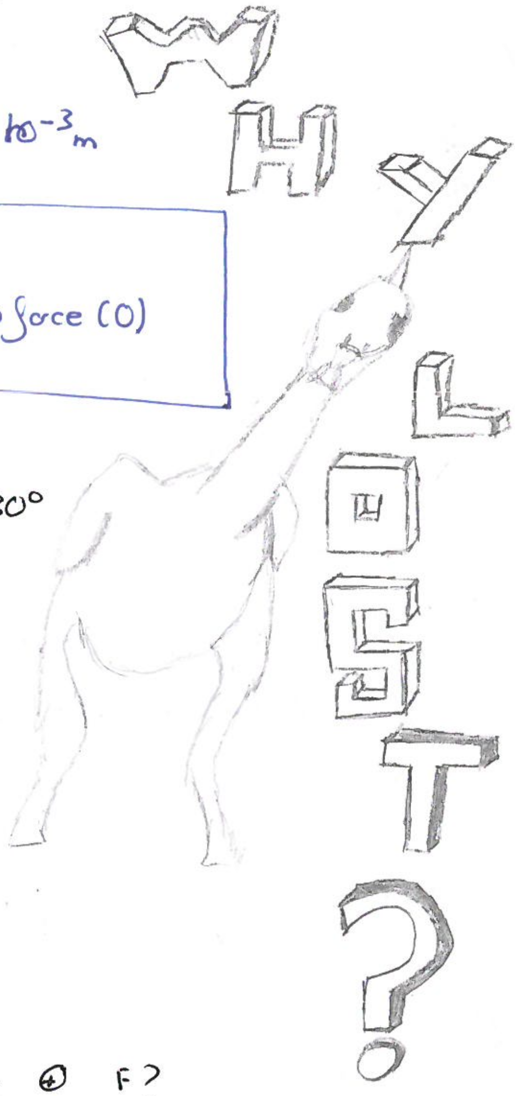


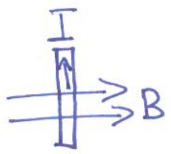
1/ $I = 45A \downarrow$ B? $R = 2\,cm$

$$B = \frac{\mu_0 \cdot I}{2\pi \cdot R} = \frac{4\pi 10^{-7} \cdot 45}{2\pi \cdot 0'02} = 4'5 \cdot 10^{-4}\,T$$

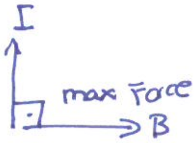
2/ $I = 10\,A$ $B = 8 \cdot 10^{-4}\,T.$ R?

$$B = \frac{\mu_0 \cdot I}{2\pi \cdot R}\ ;\ R = \frac{\mu_0 \cdot I}{2\pi \cdot B} = \frac{4\pi \cdot 10^{-7} \cdot 10}{2\pi \cdot 8 \cdot 10^{-4}} = 2'5\ 10^{-3}\,m$$





$F = I \cdot L \cdot B \cdot \sin\theta$



$\xrightarrow{I}$ $\xrightarrow{}_B$ No force (0)

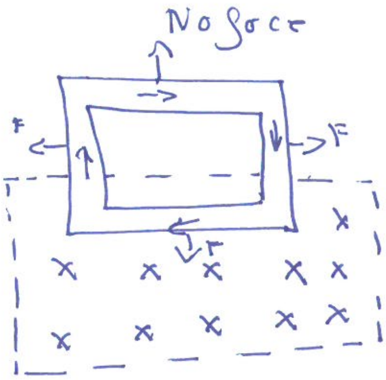
1/ $L = 2'5m$ $I = 5A$ $B = 2 \cdot 10^{-3}\,T$ F? $\theta = 30^\circ$

$F = I \cdot L \cdot B \cdot \sin\theta = 5 \cdot 2'5 \cdot 2 \cdot 10^{-3} \cdot \sin 30 = 0'0125\,N$

2/ $\overleftarrow{I} = 35\,A$ $F = 0'75\,N/m$ $B\downarrow$?

$F/L = I \cdot B \cdot \sin\theta$; ~~$I \cdot L \cdot \sin\theta$~~

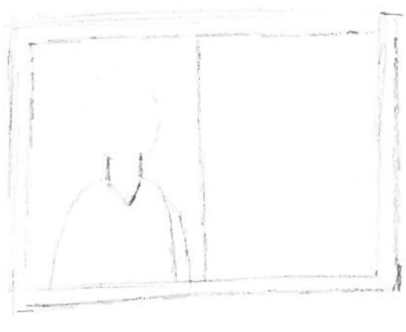
$$\frac{F}{L \cdot I \cdot \sin\theta} = B = \frac{0'75}{35 \cdot \sin 90} = 0'02148$$



$F = q \cdot v \cdot B$

1/ $\vec{v} = 4 \cdot 10^6\,m/s$ $B = 2 \cdot 10^{-4}\,T$ ⊙ F?

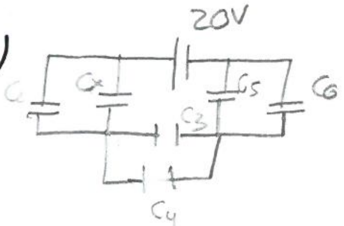
$F = q \cdot v \cdot B = 1'6 \cdot 10^{-19}C \cdot 4 \cdot 10^{6} \cdot 2 \cdot 10^{-4} = 1'28 \cdot 10^{-16}\,N$



10/ $C = 4'4 \mu F = 4'4 \cdot 10^{-6} F$ $\quad q = 3'8 mC = 3'8 \cdot 10^{-3} C$

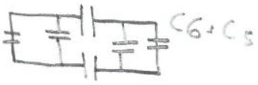
$C = q/V$; $V = q/C = 3'8 \cdot 10^{-3} / 4'4 \cdot 10^{-6} F = 863'7 V$

3/



$C_2 = 65 \mu F$; $C_3 = 24 \mu F$; $C_4 = 16 \mu F$; $C_5 = 8 \mu F$; $C_6 = 12 \mu F$

$C_{Total} = 12 \mu F$



$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{24} + \frac{1}{16} = \frac{5}{48}$; $C_{34} = 48/5 = 9'6 \mu F$

$C_{65} = \frac{1}{C_6} + \frac{1}{C_5} = \frac{1}{8} + \frac{1}{12} = \frac{5}{24}$; $C_{65} = 24/5 = 4'8 \mu F$

3/ $q_0 = -4'00 \mu C = -4 \cdot 10^{-6} C$

$W_{AB} = 0'9 mJ \rightarrow 0'9 \cdot 10^{-3} J$

$V_A = 327 V$

$W_{AB} = -q \cdot \Delta V = -q(V_B - V_a)$

$0'9 \cdot 10^{-3} = -(-4 \cdot 10^{-6})(V_B - 327)$; $0'9 \cdot 10^{-3} = 4 \cdot 10^{-6} V_B - 1'308 \cdot 10^{-3}$

$V_B = \frac{2'208 \cdot 10^{-3}}{4 \cdot 10^{-6}} = 552 V$

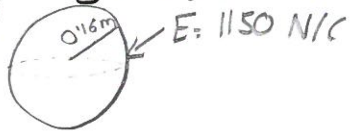
4/ $\varepsilon_1 = 2'00 \varepsilon_0$

$\varepsilon_2 = 3'00 \varepsilon_0$

$\varepsilon_3 = 6'00 \varepsilon_0$

$d = 5'00 mm$

1) Esfera aislada cerrada



a) ¿q?

$$\Phi = E \cdot S = \frac{q}{r^2} \cdot 4\pi r^2 \cdot \frac{1}{4\pi\varepsilon_0} = \frac{q}{\varepsilon_0}$$

$$E \cdot S = \frac{q}{\varepsilon_0} \; ; \; q = E \cdot S \cdot \varepsilon_0 = 1150 \cdot 4\pi \cdot 0'16^2 \varepsilon_0 = 3'27 \cdot 10^{-9} C$$

b) E a una distancia de 0'5m de su centro

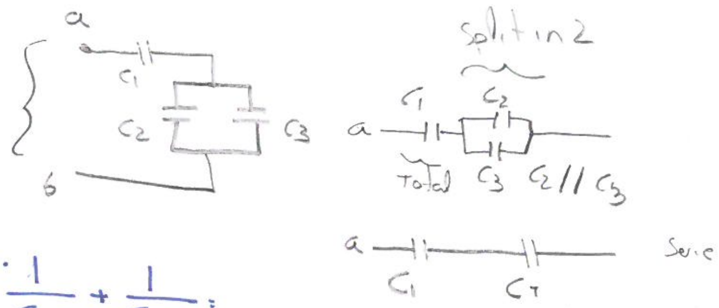
$$\Phi = E \cdot S = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{\varepsilon_0 \cdot S} = \frac{3'27 \cdot 10^{-9}}{8'85 \cdot 10^{-12} \cdot 4\pi \cdot 0'5^2} = 117'755 \; N/C$$

2) $E = \frac{\sigma}{\varepsilon_0}$; $\sigma = E \cdot \varepsilon_0 = 37'1 \cdot 10^6 \cdot 8'85 \cdot 10^{-12} = 3'3 \cdot 10^{-4}$

3) $C_2 = 6'6 \mu F$ $V = 12V$ q en C_2?

$C_3 = 6'8 \mu F$ $C_1 = 6'2 \mu F$



$C = \frac{q}{V}$; $q = C \cdot V = 12 \cdot 6'6 \mu F$

$C_{T_{32}} = 6'6 + 6'8 = 13'4 \mu F$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_{T_{34}}} =$$

$q = C_T \cdot V_T$

$$\frac{1}{C_T} = \frac{1}{6'2} + \frac{1}{13'4} = \frac{490}{2077} \; ; \; C_T = \frac{2077}{490} = 4'24 \mu F$$

$q_T = 4'24 \cdot 12 = 50'88 \mu C$

$V_1 = q_1 / C_1 = 50'88 / 6'2 = 8'2V$

$V_{2 \text{ and } 3} = V_T - V_1 = 12 - 8'2 = 3'8V$

$q_2? = Q_2 = C \cdot V = 6'6 \cdot 3'8 = 25'1$

4) $r_1 = 8'4cm = 0'084m$ $r_2 = 16'4cm = 0'164m$ ¿q? $V_{21} = 43'9V$

$$C = \frac{R_1 R_2}{R_2 - R_1} = \frac{0'084 \cdot 0'164}{0'164 - 0'084} \cdot 4\pi\varepsilon_0 = 1'91 \cdot 10^{-11}$$

$$C = q/V \; ; \; q = C \cdot V = 1'91 \cdot 10^{-11} \cdot 43'9 = 8'407 \cdot 10^{-10} C \cdot \frac{1 nC}{10^{-9} C} = 0'84071 \ldots nC$$

2/ $T_1 = 1045'3\,°C$ $T_2 = 1038'9\,°C$ $T_3 = 1041'4\,°C$ sensivity $= 0'8\,°C$

$\bar{X} = \frac{1045'3 + 1038'9 + 1041'4}{3} = 1041'87\,°C$

$T = 100 \cdot \frac{R}{\bar{X}} = 100 \cdot \frac{6'4}{1041'87} \cdot 0'61\%$

Range $= |x_{max} - x_{min}| = 1045'3 - 1038'9 = 6'4\,°C$

$T = 1041'9 \pm 2'2\,°C$ / $E_r = 0'21$ $E_r = \frac{2'2}{1041'9} \cdot 100 = 0'21\%$

$E_D = \frac{6'4}{4} = 1'6$

$E_a = \max\{\text{sensivity}, E_1, E_D\}$

$E_a = \max\{0'8, 2'23, 1'6$

$E_i = |x_i - \bar{x}|$

$E_1 = |1045'3 - 1041'87| = 3'43$

$E_2 = |1038'9 - 1041'87| = 2'96$

$E_3 = |1041'4 - 1041'87| = 0'47$

$E_{mean} = \frac{3'43 + 2'96 + 0'47}{3} = 2'23$

1/ $8'6 \cdot 10^7\, m^3$ lava ¿Mo?

$V = A_b \cdot h = 25 \cdot 50 \cdot 2'7 = 3375 \simeq 3400\, m^3$

$\frac{\text{Volcan}}{\text{Pool}} = \frac{8'6 \cdot 10^7}{3'4 \cdot 10^3} = 2'52 \cdot 10^4$; $Mo = 10^4$ pools

3/ $U(x,y,z) = 35 - 2x^2 - 4y^2 + 15e^{-z}$ Se encuentra en $P(10, 5, 0{,}5)$

Gradient

$\text{grad}\, U(x,y,z) = \frac{dU}{dx} + \frac{dU}{dy} + \frac{dU}{dz} = -4x\vec{i} - 8y\vec{j} - 15e^{-z}\vec{k}$; al contrario para acercarse

$(-4 \cdot 10, -8 \cdot 5, -15 \cdot e^{-5}) = \vec{\nabla} u(x,y,z) = (-40, -40, -12'9)$

$\frac{dU}{dx} = -4x$

$\vec{u}_{r_0}$

$\frac{dU}{dy} = -8y$

$\frac{dU}{dz} = -15e^{-z}$