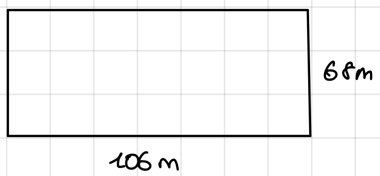
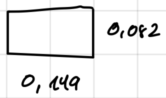
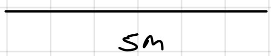
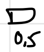
Examen 2021 Grupo I



$5,9 \cdot 10^{7}$ $\qquad 10^{8}$







2.

$C = \varepsilon_0 \frac{a\,b}{d}$ $\qquad \varepsilon_0$ without error. $= 8,8542 \cdot 10^{-12}$ F/m

$a = 5,50 \pm 0,29$ cm $\qquad b = 8.00 \pm 0,15$ $\qquad d = 0,86 \pm 0,13$ mm

$\varepsilon_r a = \frac{0,29}{5,50} = 0,0527$ $\qquad \varepsilon_r b = \frac{0,15}{8} = 0,01875$ $\qquad \varepsilon_r d = \frac{0,13 \cdot 10^{-1}}{0,86 \cdot 10^{-1}} = 0,1511$

$$\varepsilon_r(c) = \sqrt{[1 \cdot \varepsilon_r a]^2 + [1 \cdot \varepsilon_r b]^2 \cdot [\cdot 1 \cdot \varepsilon_r d]^2}$$

$$\varepsilon_r(c) = \sqrt{[0,0527]^2 + [0,018]^2 + [0,1511]^2} = 0,161189 \qquad 16\%$$

$$\varepsilon_a(C) = C \cdot \varepsilon_r C = 4,5 \cdot 10^{-11} \cdot 0,161189 = 7,3 \cdot 10^{-12}$$

$C = 4,5 \cdot 10^{-11} \cdot$

$C = 4,5 \cdot 10^{-11} \pm 7,3 \cdot 10^{-12}$

$\boxed{(45 \pm 7) \text{ pF.}}$

PREGUNTAS:

- Es necesario poner $10^{18} = 1$ Em
- $C = (45 \pm 7)$ pF o $C = (45 \cdot 10^{-12} \pm 7 \cdot 10^{-12})$
- En la 2 conseguí calcularlo con los absolutos y los relativos.
- Si calculo el Er en cm en vez de metros como Eo pasa algo??
- Hay que hacerlo con Er y Ea o solo uno
- $x = a \cdot 10^{b}$, $1 \leq a \leq 9$
- $1,15 \pm 0,015$ no es la primera pero sí la primera significant figure.

3.

$$u(x,y,z) = x^3 + y^2 + z$$

$$\vec{\nabla} u(x,y,z) = (3x^2)\vec{i} + 2y\vec{j} + 1\,\cancel{\vec{k}} = \vec{G}$$

$$\vec{\nabla} \cdot \vec{G} = 6x + 2$$

$$\vec{\nabla} \times \vec{G} = \begin{vmatrix} x & y & z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2 & 2y & 1 \end{vmatrix} = 0x + 0y + 0 - 0 - 0 - 0 = \vec{0}$$

irrotational field

TU start at 1955

$c = 299792458$ m/s

$$\begin{array}{r} 2022 \\ -1955 \\ \hline 0067 \end{array} \qquad \begin{array}{r} 45 \\ +22 \\ \hline \boxed{67} \end{array}$$

67 años

24471,75 días

2114359200 segundos.

$6{,}3 \cdot 10^{17} \Rightarrow 10^{18}$ m
order of magnitud

2. $8{,}8542 \cdot 10^{-12}$

$a = 21.3 \pm 0{,}4$ cm $\quad b = 28{,}4 \pm 0{,}7$ cm $\quad d = 3{,}02 \pm 0{,}12$ mm

$$C = A \cdot B \cdot D^{-1} \cdot \varepsilon_0 \qquad \frac{A \cdot B}{D} : \frac{-A}{D^2}$$

$$\varepsilon_r a = \frac{0{,}4}{21{,}3} \qquad \varepsilon_r b = \frac{0{,}7}{28{,}4} \qquad \varepsilon_r d = \frac{0{,}12}{3{,}02}$$

$$\varepsilon_r c = \sqrt{\left(1 \cdot \frac{0{,}4}{21{,}3}\right)^2 + \left(\frac{0{,}7}{28{,}4}\right)^2 + \left(\frac{0{,}012}{0{,}302}\right)^2} = 0{,}05 \quad 5\%$$

$$\varepsilon_a C = C \cdot \varepsilon_r c = 0{,}05 \cdot 1{,}787 \cdot 10^{-10} = 8{,}94 \cdot 10^{-12} \simeq 9 \cdot 10^{-12}$$

$$C = 1{,}787 \cdot 10^{-10}$$

$$C + \varepsilon_a = 1{,}77 \cdot 10^{-10} \pm 9 \cdot 10^{-12}$$

$$\sqrt{\left(K \frac{B}{D} \cdot 0{,}4\right)^2 + \left(K \frac{A}{D} \cdot 0{,}7\right)^2 + \left(K \cdot \frac{A \cdot B}{D^2} \cdot 0{,}12\right)^2}$$

$$K \cdot \frac{b}{d} \cdot \varepsilon_a(a) = 8{,}8542 \cdot 10^{-12} \cdot \frac{0{,}284}{0{,}00302} \cdot 0{,}004 = 3{,}33 \cdot 10^{-12}$$

$$K \cdot \frac{a}{d}\, \varepsilon_a(b) = 8{,}8542 \cdot 10^{-12} \cdot \frac{0{,}213}{0{,}00302} \cdot 0{,}007 = 4{,}3713 \cdot 10^{-12}$$

$$K \cdot \frac{a \cdot b}{d^2}\, \varepsilon_a(d) = 8{,}8542 \cdot 10^{-12} \cdot \frac{0{,}213 \cdot 0{,}284}{(0{,}00302)^2} \cdot 0{,}00012 = 7{,}04 \cdot 10^{-12}$$

$2{,}12 \cdot 10^{-13}$

K → constante

$$C = \boxed{\varepsilon_0} \frac{A \cdot B}{D}$$

$u(x,y,z) = xyz + z^2$ at the point $P(1,1,1)$ along the direction $(1,-2,1)$.

$\vec{\nabla} U = yz\,\vec{i} + xz\,\vec{j} + (xy + 2z)\,\vec{k}$

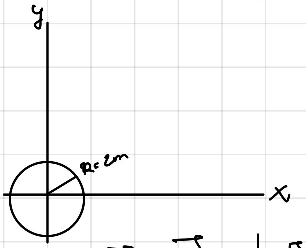
$P(1,1,1)$ $\qquad\qquad\qquad\qquad (1,1,3)$

$$n = \frac{(1,-2,1)}{\sqrt{6}} = \left(\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

$$\vec{P}\cdot\vec{n} = (1,1,3)\cdot\left(\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right) = \frac{1}{\sqrt{6}} - \frac{2}{\sqrt{6}} + \frac{3}{\sqrt{6}} =$$

$$= \frac{2}{\sqrt{6}} = \frac{\sqrt{2}}{\sqrt{3}}$$

$\vec{A} = x^3 \vec{i} + y^3 \vec{j} + z^3 \vec{k}$

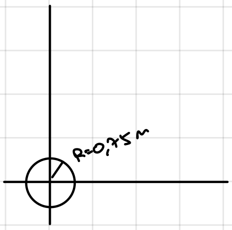


$$C\vec{A} = \oint_c \vec{A} \cdot d\vec{r} = \int_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{S}$$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3 & y^3 & z^3 \end{vmatrix} = \vec{0}$$

$$C\vec{A} = 0$$

3. $\vec{A} = 5x^3 \vec{i} - 4y^2 \vec{j} - 2z^4 \vec{k}$ $R = 0{,}75m$



$$C = \oint_c \vec{F} \cdot d\vec{r} = \oint_s (\vec{\nabla} \times \vec{F}) \cdot d\vec{s}$$

$\vec{a} \cdot \vec{b} = 8$ $\vec{a} \cdot \vec{a} = 25$ $\vec{b} \cdot \vec{b} = 16$

calculate $|a \times b|$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = a_y \cdot b_z \vec{i} + a_x \cdot b_y \vec{k} + a_z \cdot b_x \vec{j} - a_y \cdot b_x \vec{k} - b_y a_z \vec{i} - b_z \cdot a_x \vec{j}$$

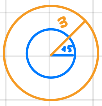
$a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z = 8$

$a_x \cdot a_x + a_y \cdot a_y + a_z \cdot a_z = 25$

$b_x \cdot b_x + b_y \cdot b_y + b_z \cdot b_z = 16$

2021. 2. C

1



$\sigma_1 = 25\,pC/m^2$ $r_1 = 0{,}015$

$\sigma_2 = 75\,pC/m^2$ $r_2 = 0{,}03$

At a point $r = 0{,}035$

$$q_1 = \sigma_1 \cdot 4\pi \cdot r_1^2 = 7 \cdot 10^{-14} \quad q_2 = \sigma_2 \cdot 4\pi \cdot r_2^2 = 8{,}5 \cdot 10^{-13}$$

$$E = \frac{q_1 \cdot}{4\pi \cdot r^2 \cdot \varepsilon_0} + \frac{q_2 \cdot}{4\pi \cdot r^2 \cdot \varepsilon_0} = 0{,}513 + 6{,}239 \qquad = 6{,}75\ N/C$$

2

$$W_{AB} = -q \cdot \Delta V = +8 \cdot 10^{-3} \cdot (50 - 36) = 0{,}112\ mJ$$

$$W_{DC} = -q \cdot \Delta V = 8 \cdot 10^{-3} \cdot (36 - 36) = 0\ J$$

3.

$$\frac{1}{C_{eq}} = \frac{1}{C_{x2}} + \frac{1}{C_{34}} + \frac{1}{C_{56}} \rightarrow \frac{1}{C_{eq}} - \frac{1}{C_{34}} - \frac{1}{C_{56}} = \frac{1}{C_{x2}} \rightarrow$$

$$\rightarrow \frac{1}{12} - \frac{1}{40} - \frac{1}{20} = \frac{1}{C_{x2}} \rightarrow \frac{10}{120} - \frac{3}{120} - \frac{6}{120} = \frac{1}{C_{x2}} \rightarrow C_{x2} = 120\,\mu F$$

$$C_x = 120 - C_2 \rightarrow C_x = 55\,\mu F$$

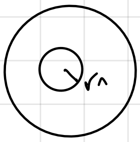
$$Q_{eq} = C_{eq} \cdot V_{eq} = 12\,\mu F \cdot 20 = 2{,}4 \cdot 10^{-4}\,C = Q_{12}$$

$$V_{x2} = \frac{Q_{x2}}{C_{x2}} = \frac{2{,}4 \cdot 10^{-4}}{120 \cdot 10^{-6}} = 2$$

$$U_x = \frac{1}{2} \cdot C_x \cdot V_x^2 = \frac{1}{2} \cdot 55\,\mu F \cdot (2)^2 = 1{,}1 \cdot 10^{-4} =$$

$$= 0{,}11\ mJ$$

2020 - 2.2.



Q and R

$r_1 = 40$ cm $\quad E_1 = 66{,}67$ V/m Inside

$r_2 = 80$ cm $\quad E_2 = 56{,}25$ V/m Inside/outside?

$$E = \frac{q_{\text{encerrada}}}{4\pi\varepsilon_0 r^2} \Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E_2 \cdot 4\pi \cdot \varepsilon_0 \cdot r_2^2 = Q$$

si $r \leq R \Rightarrow \quad E = \dfrac{Q \cdot r}{4\pi\varepsilon_0 \cdot R^3}$

$$E_1 = \frac{Q \cdot r_1}{4\pi \cdot \varepsilon_0 \cdot R^3} \Rightarrow \frac{E_1 \cdot 4\pi \cdot \varepsilon_0 \cdot R^3}{r_1} = Q = 4 \cdot 10^{-9}$$

$$R = \sqrt[3]{\frac{E_2 \cdot r_2^2 \cdot r_1}{E_1}} \qquad E_2 \cdot \cancel{4\pi} \cdot \cancel{\varepsilon_0} \cdot r_2^2 = \frac{E_1 \cdot \cancel{4\pi} \cdot \cancel{\varepsilon_0} \cdot R^3}{r_1}$$

$[R = 0{,}6 \text{ m}]$

$$E_{(R)} = \frac{4 \cdot 10^{-9}}{4\pi \cdot \varepsilon_0 \cdot R^2} = 100 \text{ V/m}$$

2. $q_0 = -4\ \mu C \quad W_{AB} = 0{,}9$ mJ $\qquad$ Potential of A is 327 V

$$W_{AB} = -q \cdot (V_B - V_A) \Rightarrow \frac{W_{AB}}{-q} = (V_B - V_A) \Rightarrow \frac{W_{AB}}{-q} + V_A = V_B$$

$$V_B = \frac{0{,}9 \cdot 10^{-3}}{4 \cdot 10^{-6}} + 327 = [552 \text{ V}]$$

3

$\varepsilon_1 = 2.00\,\varepsilon_0 \qquad \varepsilon_2 = 3.00\,\varepsilon_0 \qquad \varepsilon_3 = 6.00\,\varepsilon_0 \qquad d = 6.00\ mm \qquad A = 444\ cm^2$

$6 \cdot 10^{-3} \qquad 444 \cdot 10^{-4}$

$$C_1 = \frac{2 \cdot \varepsilon_0 \cdot A}{d} = \frac{2 \cdot 8{,}85 \cdot 10^{-12} \cdot 444 \cdot 10^{-4}}{2 \cdot 10^{-3}} = 3{,}9 \cdot 10^{-10}$$

$$C_2 = 5{,}9 \cdot 10^{-10}$$

$$C_3 = 1{,}17 \cdot 10^{-9}$$

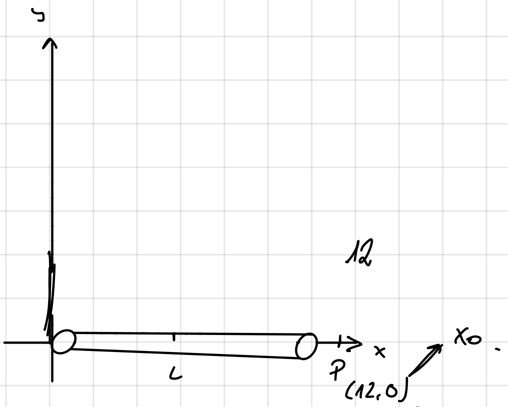
$$\frac{1}{C_{ef}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{ef}} = \frac{1}{3{,}9 \cdot 10^{-10}} + \frac{1}{5{,}9 \cdot 10^{-10}} + \frac{1}{1{,}17 \cdot 10^{-9}} = \frac{7 \cdot 10^{-19} + 4{,}6 \cdot 10^{-19} + 2{,}3 \cdot 10^{-19}}{2{,}7 \cdot 10^{-28}}$$

$$C_{ef} = \frac{2{,}7 \cdot 10^{-28}}{1{,}4 \cdot 10^{-18}} = 1{,}9 \cdot 10^{-10} = 190\,pF$$

1-

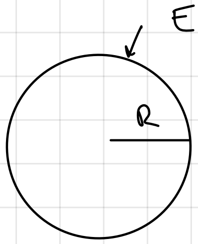
duda.



$q = 72 nC$ $\qquad$ $E = 6\ N/C$

2.

$E = 1{,}5\ MV/m$

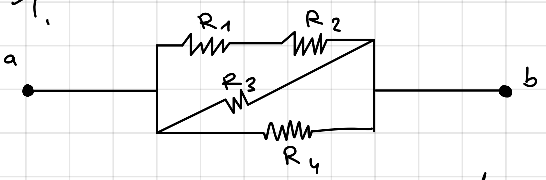


V en su centro es 550 kV

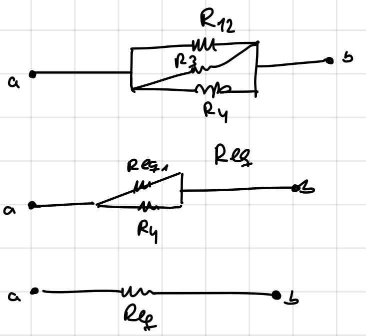
$V =$

2021.3.I

1.



$R = 40\,\Omega$ $\qquad R = \frac{V}{I}$



$R_{12} = 80\,\Omega$

$R_3 = 40\,\Omega$

$R_4 = 40\,\Omega$

$R_{eq1} = \left(\frac{1}{R_{12}} + \frac{1}{R_3}\right)^{-1}$

$R_{eq1} = \left(\frac{3}{80}\right)^{-1} = \frac{80}{3}$

$R_{eq} = \left(\frac{1}{R_{eq1}} + \frac{1}{R_4}\right)^{-1}$

$R_{eq} = \left(\frac{3}{80} + \frac{1}{40}\right)^{-1} = \left(\frac{5}{80}\right)^{-1} = \frac{80}{5} = 16\,\Omega.$

2.

$v = \frac{r \cdot q \cdot B}{m}$ $\quad \boxed{\cdot 1{,}2}$

$m = 1{,}5 \cdot 10^{-3}\ Kg$

$q = 0{,}5\ C.$

$B = 9{,}42\ T$

$\frac{r \cdot q \cdot B}{m} = v = 125{,}6$

3.

$a = 2\ cm$

$b = 4\ cm$

$I = 2\ A$

$B_a = \frac{\mu_0 \cdot I \cdot \pi/2}{2R\,2\pi}\ \hat{i}$

$B_b = \frac{\mu_0 \cdot I \cdot 3\pi/2}{2R \cdot 2\pi}$

$B_{TOTAL} = B_a + B_b$

$1{,}57 \cdot 10^{-5} + 2{,}35 \cdot 10^{-5}$

$= 4 \cdot 10^{-5}\ \hat{i}$

2021.3.B.

1.

$$\begin{pmatrix} 30 & -20 \\ -20 & 30 \end{pmatrix} \begin{pmatrix} I1 \\ I2 \end{pmatrix} = \begin{pmatrix} 12 \\ -15 \end{pmatrix}$$

$$I1 = \frac{\begin{vmatrix} 12 & -20 \\ -15 & 30 \end{vmatrix}}{\begin{vmatrix} 30 & -20 \\ -20 & 30 \end{vmatrix}} = \frac{60}{500} = 0,12$$

$$I2 = \frac{\begin{vmatrix} 30 & 12 \\ -20 & -15 \end{vmatrix}}{\begin{vmatrix} 30 & -20 \\ -20 & 30 \end{vmatrix}} = \frac{-210}{500} = -0,42$$

$$i_3 = I1 - I2 = 0,54 A$$

2.

$$\boxed{B = 3\vec{j} + 5\vec{k}\ T}$$

0

$q = 2C$

$v = 2\vec{i} + 4\vec{j} + 6\vec{k}$

$B = \quad By\,\vec{j} + Bz\,\vec{k}$

$\vec{F} = 4\vec{i} - 20\vec{j} + 12\vec{k}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 4 & 6 \\ 0 & By & Bz \end{vmatrix}$$

$$\vec{F} = q \cdot \vec{v} \times \vec{B}$$

$$(4\vec{i} - 20\vec{j} + 12\vec{k}) = 2 \cdot (2\vec{i} + 4\vec{j} + 6\vec{k}) \times (By\vec{j} + Bz\vec{k})$$

$$(4\vec{i} - 20\vec{j} + 12\vec{k}) = 2 \cdot (4Bz\,\vec{i} + 2By\,\vec{k} - 6By\,\vec{i} - 2Bz\,\vec{j})$$

$$(4\vec{i} - 20\vec{j} + 12\vec{k}) = 2 \cdot (4Bz - 6By\,\vec{i} - 2Bz\,\vec{j} + 2By\,\vec{k})$$

$$(4\vec{i} - 20\vec{j} + 12\vec{k}) = (8Bz - 12By\ \vec{i} - 4Bz\,\vec{j} + 4By\ \vec{k})$$

$By = 3$

$Bz = 5$

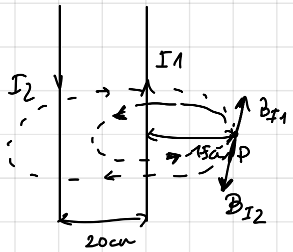
3.

$d = 20\ cm$

$I_1 = 30\ A$

$I_2 = 40\ A$

$P = 15\ cm$



$$B_1 = \frac{\mu \cdot I_1}{2\pi \cdot d_1} = -\frac{\mu \cdot 30}{2\pi \cdot (15 \cdot 10^{-2})}\,\vec{i} = -4 \cdot 10^{-5}\,\vec{i}$$

$$B_2 = \frac{\mu \cdot I_2}{2\pi \cdot d_2} = +\frac{\mu \cdot 40}{2\pi \cdot (35 \cdot 10^{-2})}\,\vec{i} = 2,3 \cdot 10^{-5}\,\vec{i}$$

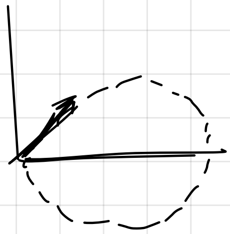
$$B_{TOTAL} = B_1 + B_2 = -1,7 \cdot 10^{-5}\,\vec{i}\ T$$

electron

$(0,0)$ $v = 3,8 \cdot 10^{6}$ m/s

$B = 9,6 \cdot 10^{-3}$

$m = 9,11 \cdot 10^{-31}$ $e = 1,6 \cdot 10^{-19}$ C.



$$R = \frac{m \cdot v}{q \cdot B} = 2,25 \cdot 10^{-3}$$

$$\omega = \frac{q \cdot B \cdot C}{m \cdot v} = 1$$

$$f = \frac{1}{2\pi} \qquad T = \frac{2\pi m}{|q| \cdot B} = 3,7 \text{ nS}$$

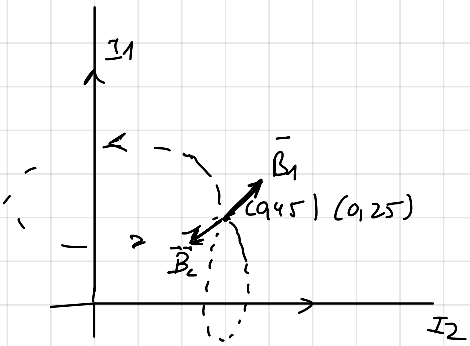
2.

$E = 1,42 \cdot 10^{3}$ V/m $\qquad B = 5,7$T

$$v = \frac{E}{B} = 250 \text{ m/s}$$

3.

$I = 3,5$ A $\qquad (0,45, 0,25)$



$$B_1 = \frac{\mu_0}{2\pi} \cdot \frac{I_1}{d_1} = \frac{\mu_0 \cdot 3,5}{2\pi \cdot (0,45)} = -1,5 \cdot 10^{-6}\,\vec{k}$$

$$B_2 = 2,8 \cdot 10^{-6}$$

$$B_{TOTAL} = B_2 + B_1 = 1,2 \cdot 10^{-6}\,\vec{k}\,T$$

4.

speed $v = 0{,}32$ m/s
$d = 5{,}4$ mm $\rightarrow 5{,}4 \cdot 10^{-3}$
$B = 0{,}65$ T

$B = \frac{\mu}{2\pi} \cdot \frac{I}{x}$ $\qquad \mathcal{E} = B \cdot L \cdot v = (0{,}65)(5{,}4 \cdot 10^{-3})(0{,}32) = 1{,}123 \cdot 10^{-3}$ V

5. SI : $\Phi_B(t) = N \cdot (-5{,}6t^2 + 4{,}1t - 5{,}5) \cdot 10^{-3}$ Wb Calculate $t = 2{,}4$ s

$\mathcal{E} = -N \cdot (-11{,}2t + 4{,}1) \cdot 10^{-3}$

$\mathcal{E} = -522 \cdot (-22{,}78) \cdot 10^{-3} = 12$ V

2021.3.Δ

1.
$$\begin{pmatrix} 20 & -10 \\ -10 & 30 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 20 \\ -30 \end{pmatrix}$$

$$I_1 = \frac{\begin{vmatrix} 20 & -10 \\ -30 & 30 \end{vmatrix}}{\begin{vmatrix} 20 & -10 \\ -10 & 30 \end{vmatrix}} = \frac{3\cancel{00}}{5\cancel{00}}$$

$$I_2 = \frac{\begin{vmatrix} 20 & 20 \\ -10 & -30 \end{vmatrix}}{\begin{vmatrix} 20 & -10 \\ -10 & 30 \end{vmatrix}} = -\frac{4\cancel{00}}{5\cancel{00}}$$

$$V_a - V_b = \mathcal{E} - I \cdot r = 30 - \frac{7}{5} \cdot 10 = 16$$

2.

$$\frac{3}{5} + \frac{4}{5}$$

$$I_1 = I_2 + I_3$$

$$\frac{4}{5} + \frac{3}{5} = I_3 = \frac{7}{5}$$

2.

$t = 0{,}63 \cdot 10^{-6}\ s$

$e = 1{,}6 \cdot 10^{-19}\ C$

$m = 9{,}11 \cdot 10^{-31}\ kg$

$r = 2m$

$$T = 4 \cdot 6{,}3 \cdot 10^{-5} = 2{,}52 \cdot 10^{-4}$$

$$f = 3968.2539$$

$$\omega = 2\pi \cdot f = 24933$$

$$B = \frac{\omega\, m}{q} = \frac{24933{,}28 \cdot 9{,}11 \cdot 10^{-31}}{1{,}6 \cdot 10^{-19}} = 1{,}42 \cdot 10^{-7}\ T$$

3.

$$B = \frac{\mu_0 \cdot I}{2a} \cdot n$$

$$B = 5{,}9 \cdot 10^{-5}\ T$$

$$\frac{2}{5} = l$$

$$l = 2\pi r$$

$$\boxed{0{,}063} = r$$