Limits, derivatives and Taylor polynomials

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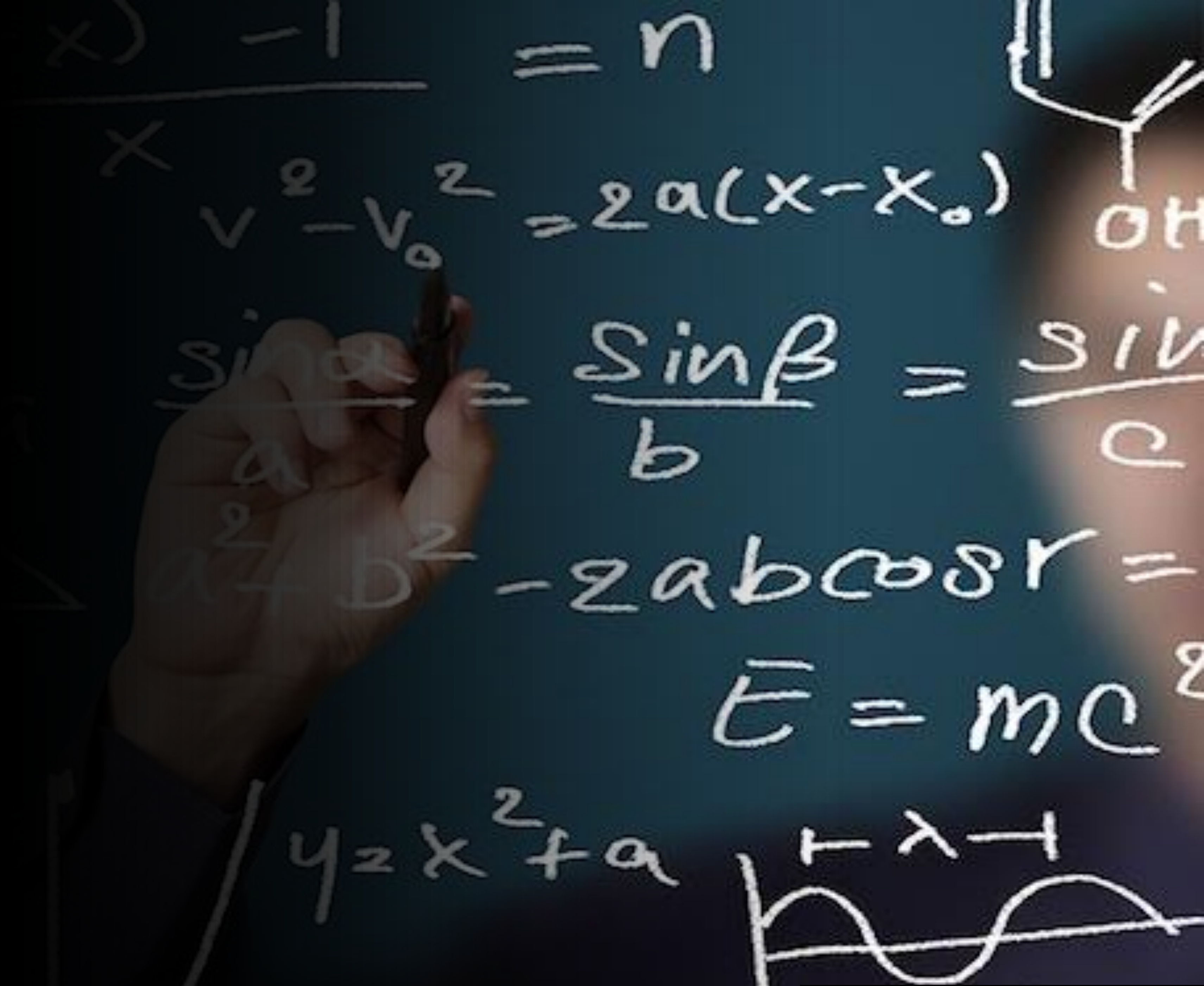


Introduction

Today we will use MATLAB to compute limits, derivatives and Taylor polynomials.

Please remember that the Symbolic Math Toolbox add-on must be installed to perform these calculations.

Functions

To create a function in MATLAB you first have to declare it as a symbolic function using the following code:

```
syms f(x)
```

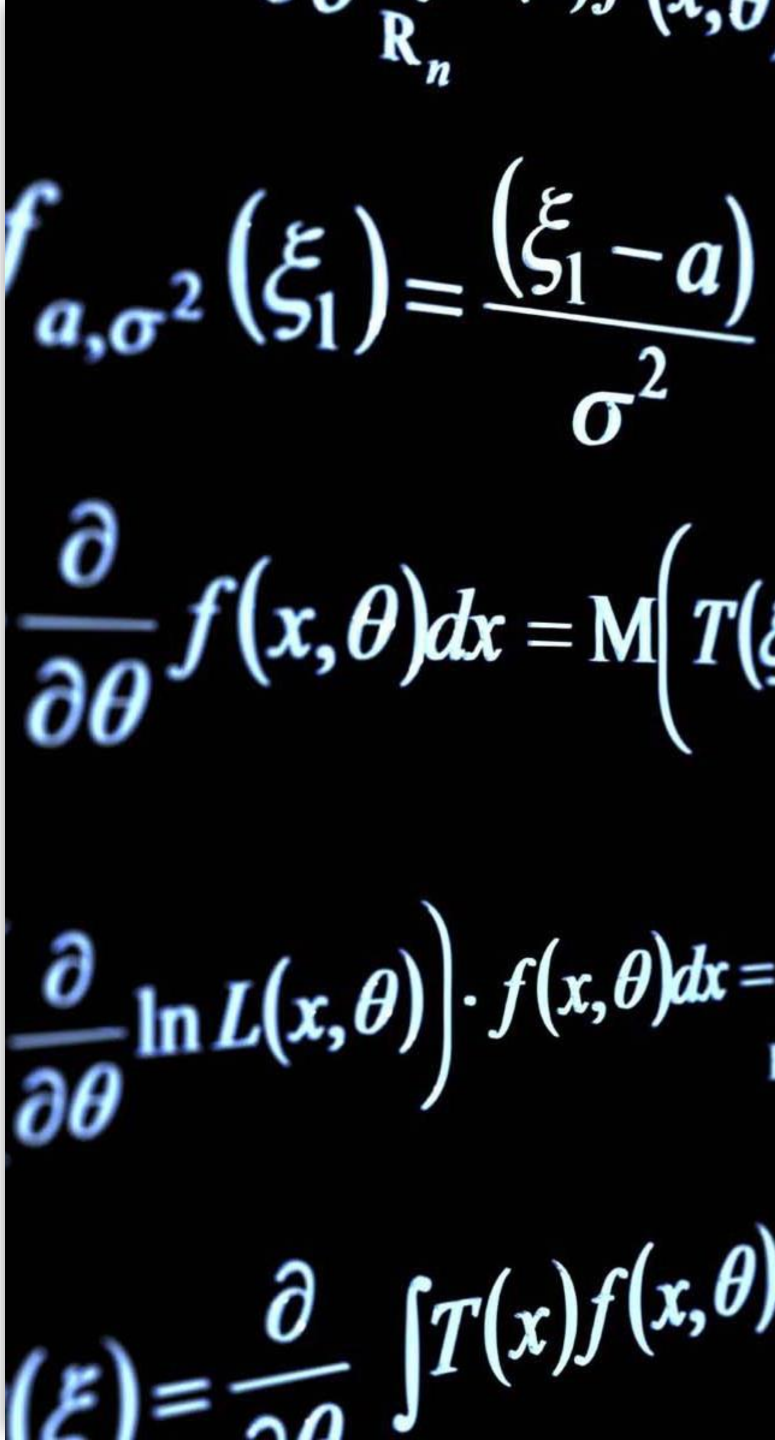
After that we can assign the value that we want that function to have. For example:

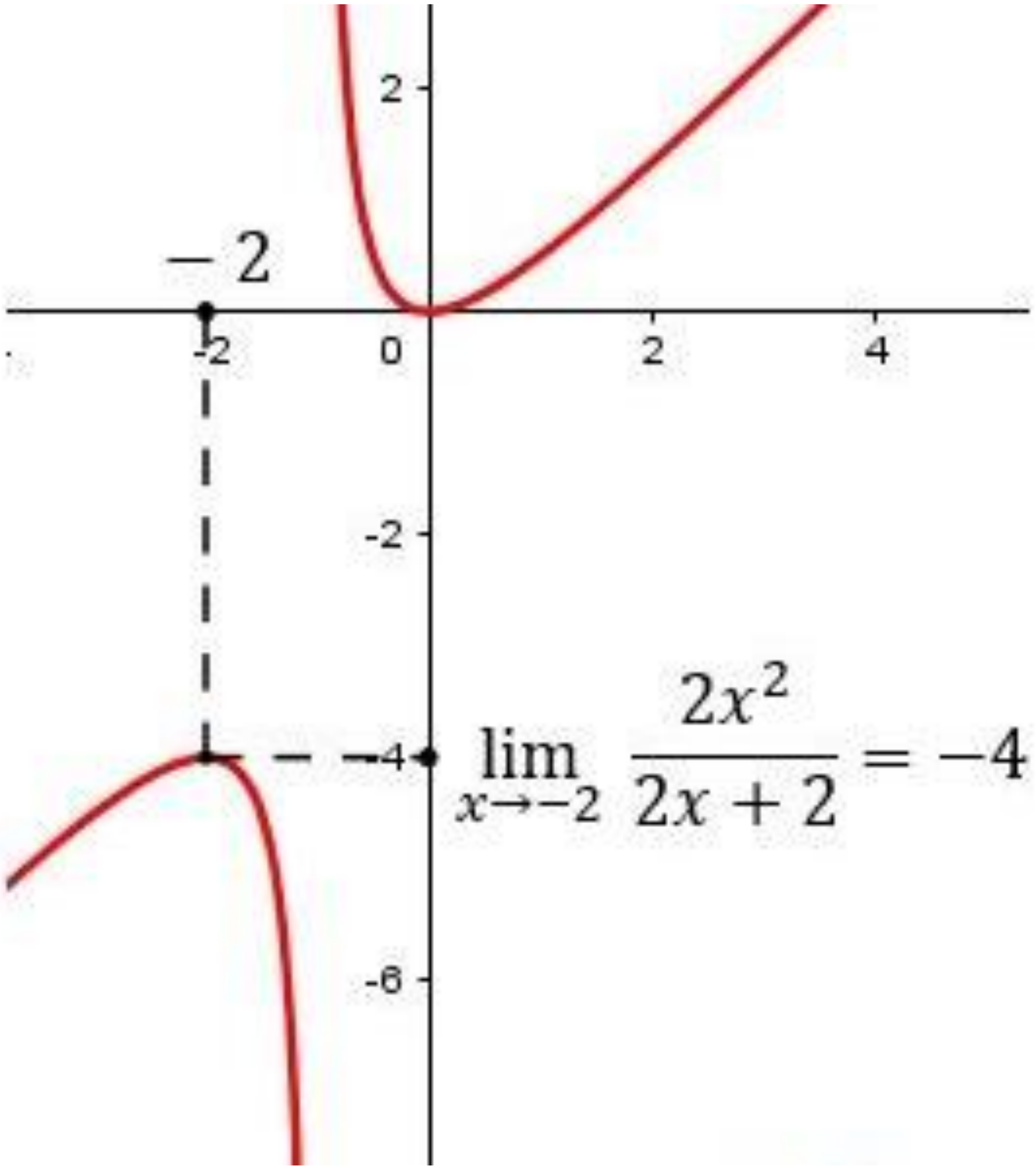
```
f(x) = sqrt(x) + 2*x – sin(x)
```

After that we can create a graph with the following code:

```
fplot(f, [0, 10])
grid on <-- Draw "the lines"
```

It's important having the Symbolic Math Toolbox add-on installed, otherwise it won't work.





Limits

To compute a limit $\lim_{x \to a} f(x)$ we have to write:

limit(f(x), x, a)

We should add a **"left"** or **"right"** if we want a side limit. In case we want x to tend to infinite we must write either **"inf"** or **"-inf"** as shown:

limit(f, x, a, ’left’) / limit(f, x, a, 'right')

limit(f(x), x, inf) / limit(f(x), x, -inf)

EXERCISE 1

- **Compute the following limits and plot the graph of the functions over an appropriate interval to show their behaviour of close to the point:**

a) $\lim_{x \to 0^+} \sqrt{x} \cdot \log(x)$

b) $\lim_{x \to 0^-} \frac{1}{x}.$

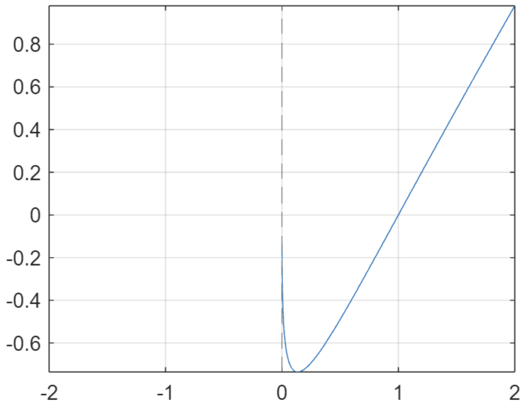
c) $\lim_{x \to +\infty} \arctan(x)$

SOLUTION

a) $\lim_{x \to 0^+} \sqrt{x} \cdot \log(x)$

```
f(x) = sqrt(x) * log(x);
a = limit(f(x), x, 0, "left")
fplot(f,[-2,2]), grid on
```

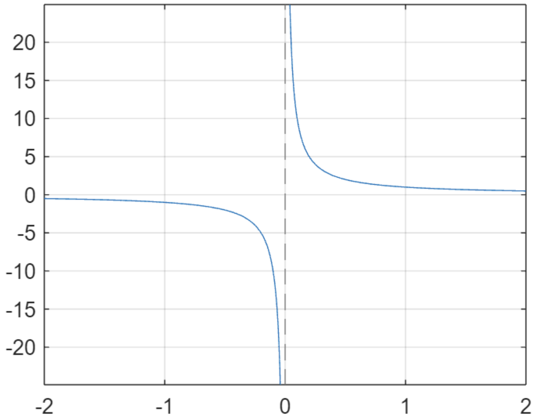
a = 0



b) $\lim_{x \to 0^-} \frac{1}{x}$

```
f(x) = 1/x;
b = limit(f(x), x, 0, "right")
fplot(f,[-2,2]), grid on
```

b = ∞

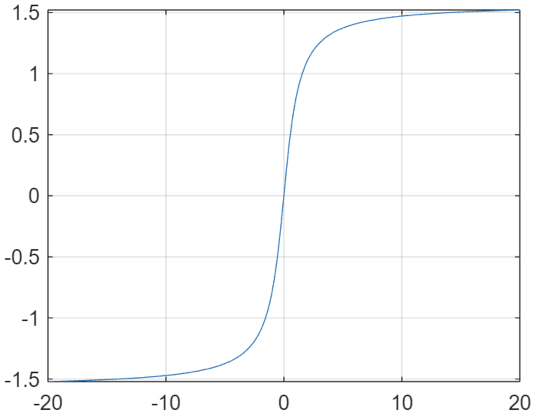


c) $\lim_{x \to +\infty} \arctan(x)$

```
f(x) = atan(x);
c = limit(f(x), x, inf)
fplot(f,[-20,20]), grid on
```

c =

$\frac{\pi}{2}$



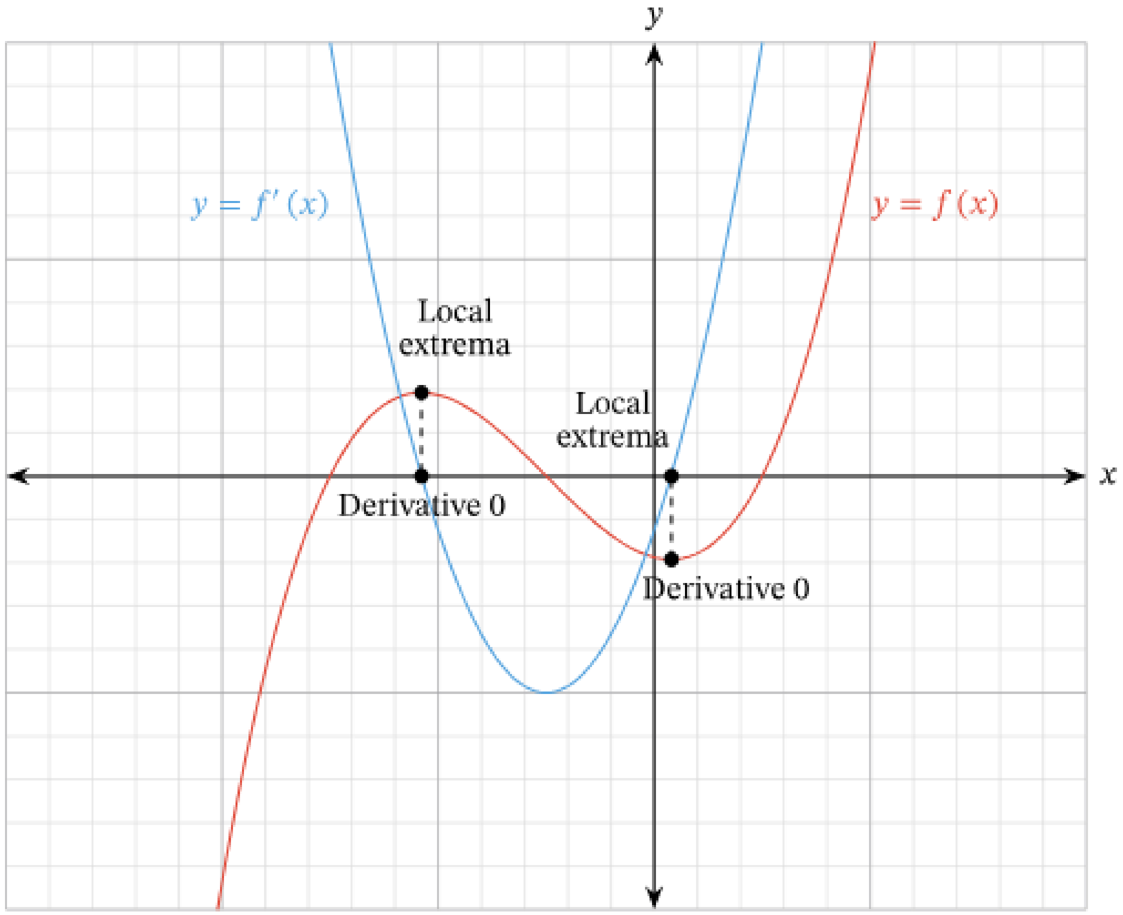
Derivatives

To compute a derivative we need to define a symbolic function and write the following expression, being n the n^{th} derivative:

Diff(f(x), x, n)

Graphically we can represent both functions and recognize which is each one with an important fact: the derivative always crosses the x axis between the minimum and the maximum of the original, meaning that:

- The derivative changes from a **negative** to a **positive** value where the function has a **minimum**.
- The derivative changes from a **positive** to a **negative** value where the function has a **maximum**.



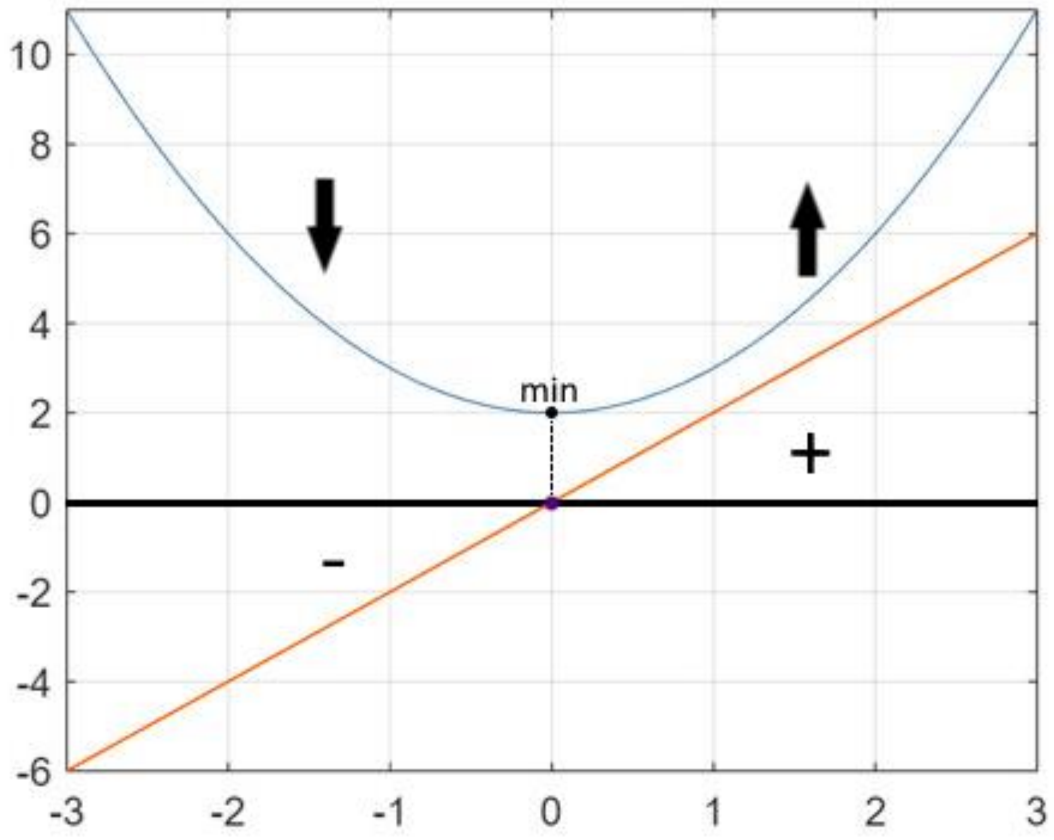
EXAMPLE

For the function f(x) = x^2 + 2 to get the derivative we should type:

```
syms f(x)
f(x)= x^2 + 2
dx(x)= diff(f(x),x)
fplot({f(x),dx}, [-3,3]), grid
```

f(x) = $x^2 + 2$

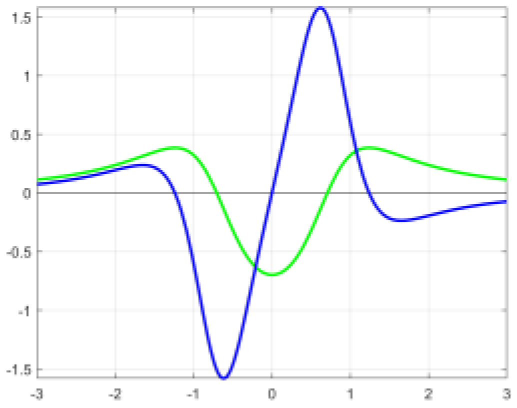
dx(x) = $2\,x$



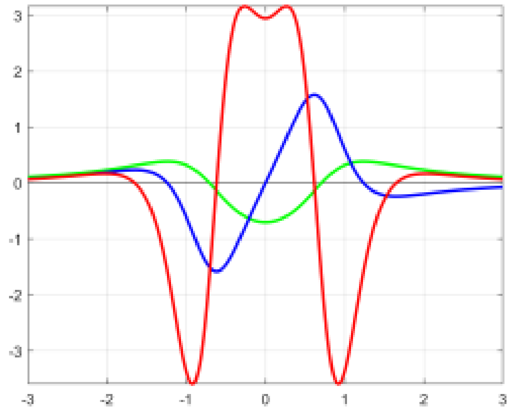
EXERCISE 2

- **Guess which is the function and which are the derivatives:**

a)

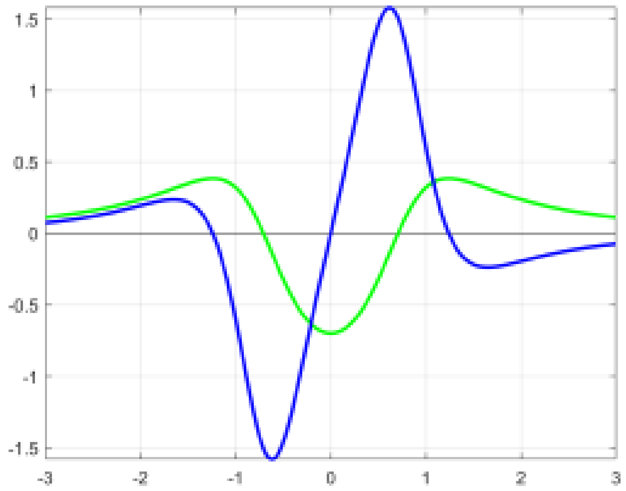


b)

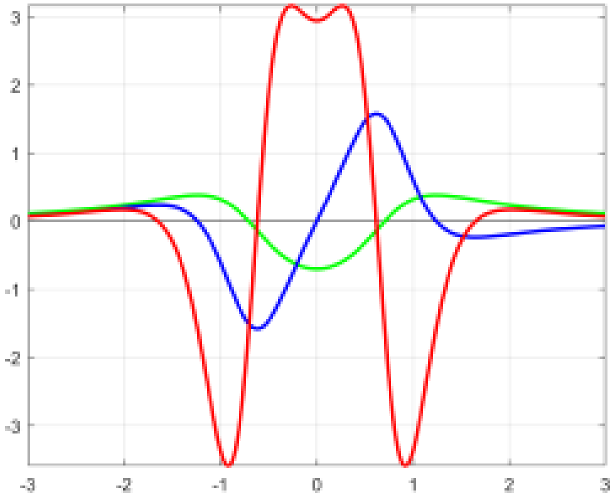


SOLUTION

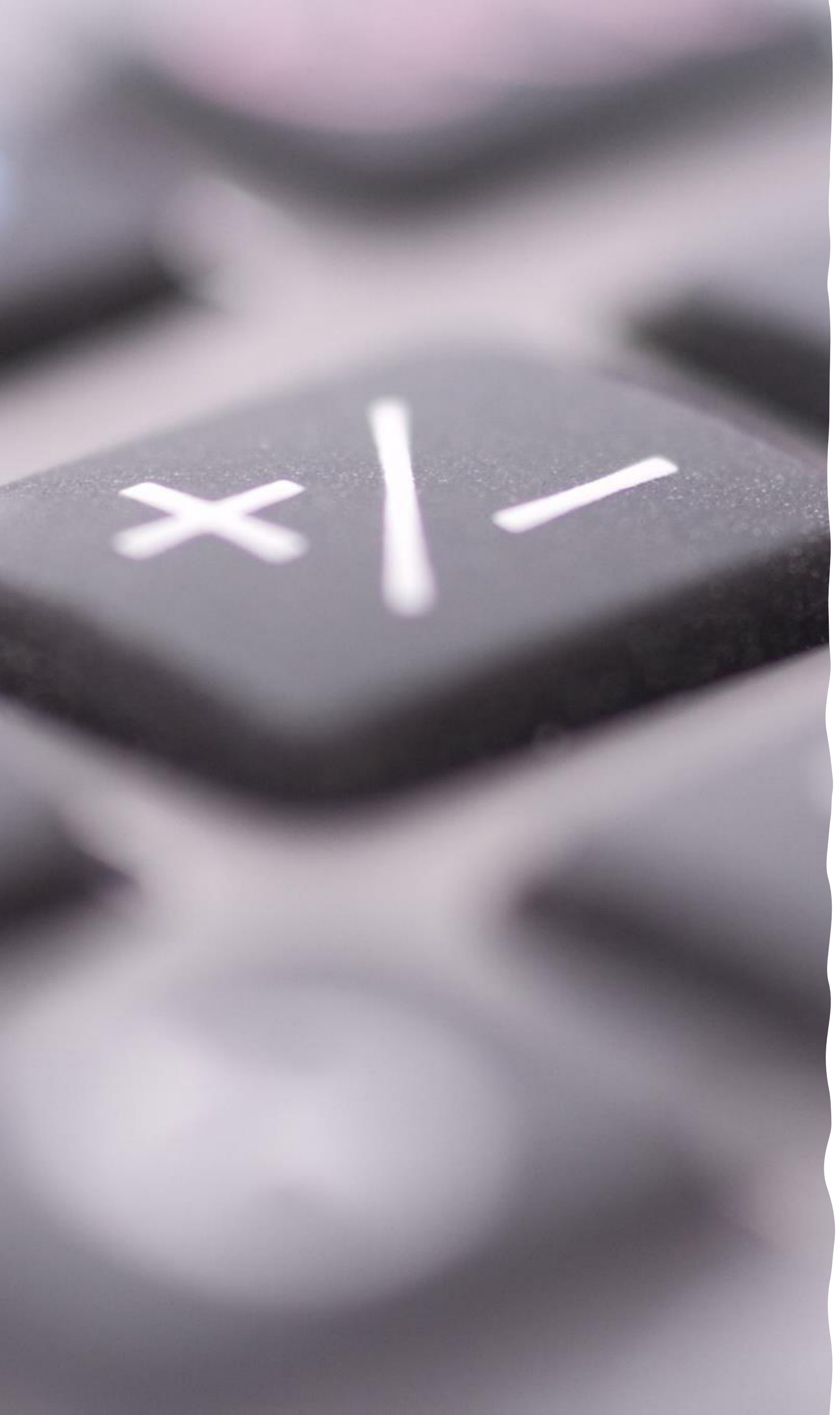
a) The green function is f(x) and the blue function is f'(x).



b) The green function is f(x), the blue function is f'(x) and the red function is f''(x).



You can see that f(x) and f'(x) are the same in both graphs. In case of having more than two functions you have to compare several pairs until you get which one is each.



Taylor's formula

To get the Taylor polynomial of '*f*' of '*n*' order around '*a*' we type:

taylor(f(x), x, a, 'Order', n+1)

<u>IMPORTANT!</u>

To get the n^{th} order polynomial we must write **n+1** as the final parameter. This is just because of the programmation of Matlab.

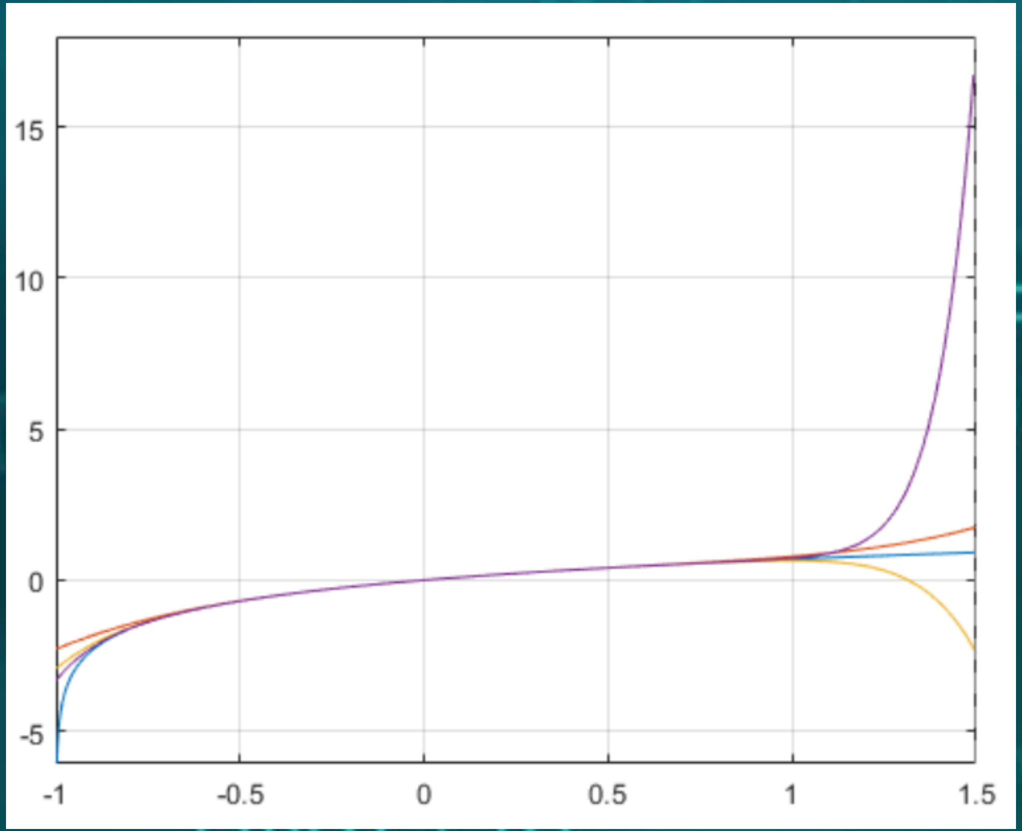
EXAMPLE

For the function f(x) = log(1 + x) to get the Taylor polynomials of orders n = 5, 10 and 15 around 0 we should type:

```
syms f(x), f(x) = log(1 + x)
tf5(x) = taylor(f(x), x, 'Order', 6)
tf10(x) = taylor(f(x), x, 'Order', 11)
tf15(x) = taylor(f(x), x, 'Order', 16)
```

In order to get the graph we should type:

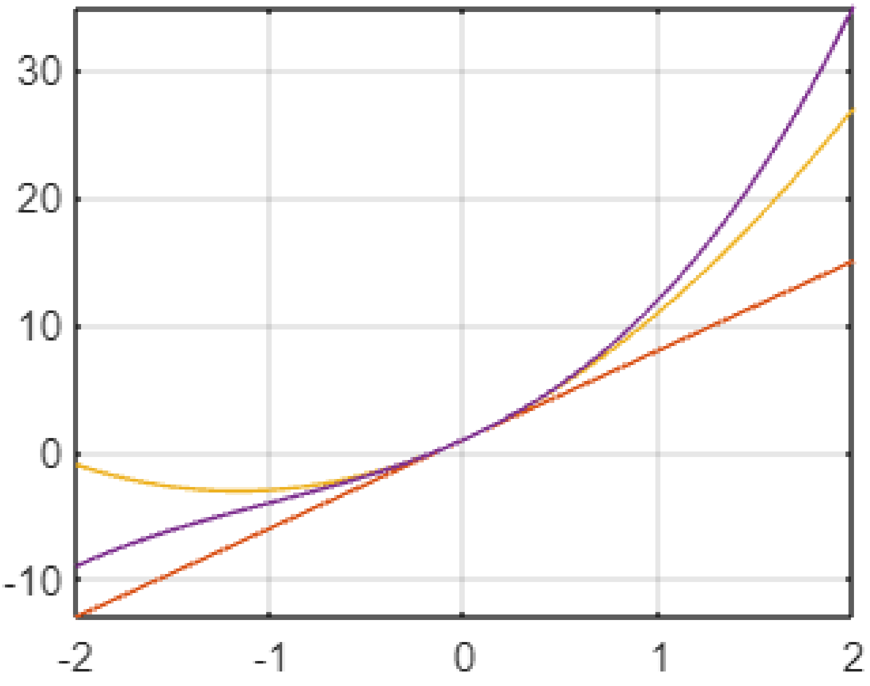
```
fplot({f, tf5, tf10, tf15}, [-1, 1.5]), grid on
```



EXERCISE 3

- **We consider the function $f(x) = x^3 + 3x^2 + 7x + 1$.**
  a) *Find the Maclaurin polynomials of orders 1, 2, 3, 4, … at $a = 0$ and show that there are only three different ones.*
  b) *Plot the function and all the different polynomials found over $[-2, 2]$. Do you recognize the polynomial of order 1?*

SOLUTION



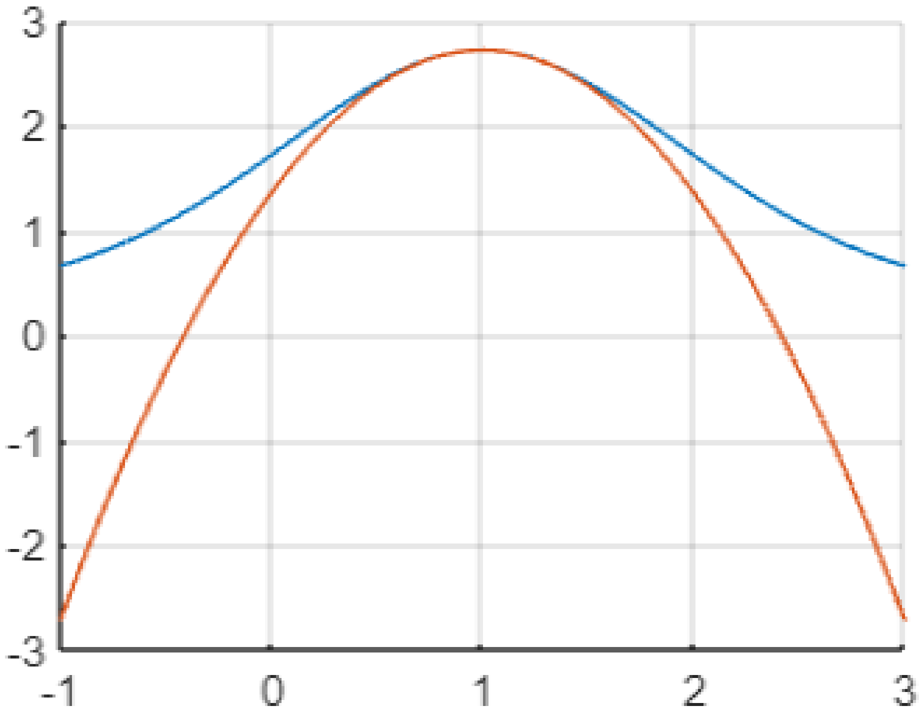

```
syms f(x), f (x) = x^3 + 3*x^2 + 7*x + 1
tf1(x)=taylor(f(x),x,'Order',2)
tf2(x)=taylor(f(x),x,'Order',3)
tf3(x)=taylor(f(x),x,'Order',4)
fplot({f, tf1, tf2, tf3}, [-2,2]), grid on
```

Order 1 is the red function, order 2 the yellow function and order 3 the purple function

EXERCISE 4

- **We consider the function $f(x) = e^{\cos(x-1)}$.**
  a) *Find the Taylor polynomial of order 3 of $f(x)$ around 1.*
  b) *Plot the function and its polynomials together over [-1, 3].*

SOLUTION

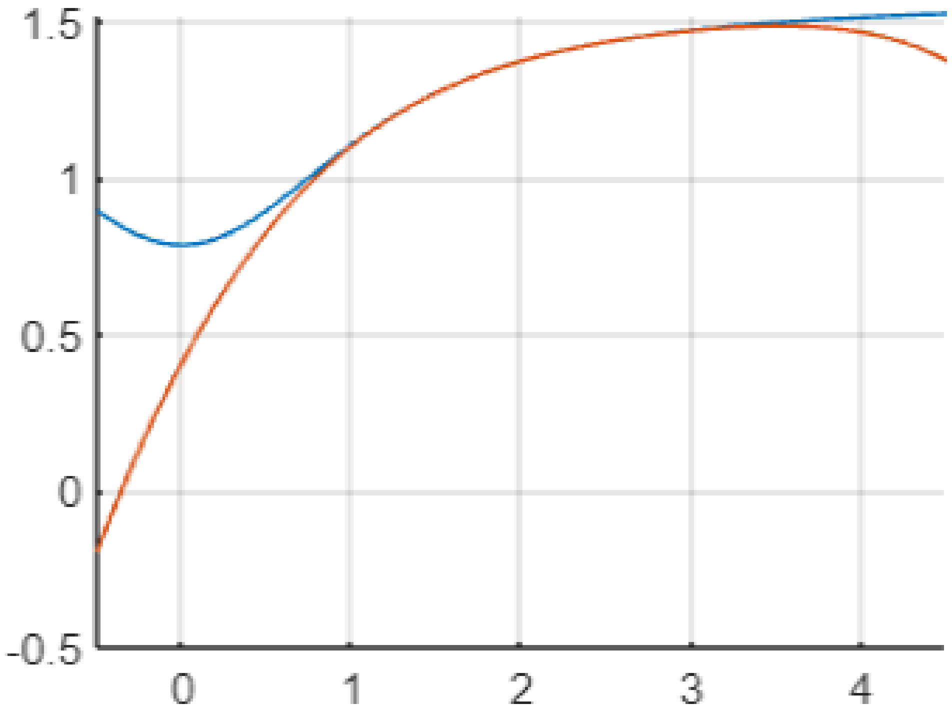



```
f(x) = exp(cos(x - 1));
tf3(x) = taylor(f(x), x, 1, "Order", 4);
hold on
fplot(f,[-1,3]), grid on
fplot(tf3, [-1, 3]), grid on
```

EXERCISE 5

- **We consider the function $f(x) = \arctan(x^2 + 1)$.**
  a) *Find the Taylor polynomial of order 5 of $f(x)$ around 2.*
  b) *Plot the function and its polynomials together over [-0.5, 4.5].*

SOLUTION




```
f(x) = atan(x^2 + 1);
tf5(x) = taylor(f(x), x, 2, "Order", 6);
hold on
fplot(f,[-0.5,4.5]), grid on
fplot(tf5, [-0.5, 4.5]), grid on
```